CONTROLLING CREST VERTICAL CURVATURE RATES BASED ON VARIABLE GRADE STOPPING SIGHT DISTANCE CALCULATION

Stergios Mavromatis, Assistant Professor
Technological Educational Institute of Athens
School of Civil Engineering and Surveying & Geoinformatics Engineering
2 Agiou Spiridonos Str.
GR-12210 Athens, Greece
Tel: (+30)2105385332
Fax: (+30)2105385317
e-mail: stemavro@teiath.gr

Nikiforos Stamatiadis, Professor
University of Kentucky
265 Raymond Bldg.
Lexington, KY 40506-0281
Tel: 859.257.8012
Fax: 859.257.4404
e-mail: nick.stamatiadis@uky.edu

Basil Psarianos, Professor
National Technical University of Athens
School of Rural & Surveying Engineering
9 Hiroon Polytechniou Str.
GR-15780 Athens, Greece
Tel: (+30)2107722628
Fax: (+30)2107722629
e-mail: psari@survey.ntua.gr

George Yannis, Professor
National Technical University of Athens
School of Civil Engineering
5 Hiroon Polytechniou Str.
GR-15773, Athens, Greece
Tel.: (+30)2107721380
Fax: (+30)2107721454
e-mail : geyannis@central.ntua.gr

Total number of words 6533
(2 Tables=500, 10 Figures=2500, 3533 Text)
Submitted to the Transportation Research Board

November 2014

ABSTRACT

Stopping Sight Distance (SSD) is a key control element which directly affects the suggested values of crucial road design parameters. Although there is a significant difference in SSD values between upgrades and downgrades, many Design Policies ignore the grade effect during vehicle braking on variable grades. Such a case is experienced during the determination of crest vertical curvature rates where the relevant SSD values are extracted assuming leveled road geometry. The paper investigates possible deficiency of this approach, regarding cases where the length of the vertical curve exceeds the control SSD values.

The authors addressed the SSD calculation on variable grades during the braking process through a recently developed process that relates the point mass model and the laws of mechanics.

For a wide range of design speed values, charts illustrating the required SSDs were drawn as a function of negative ending grade values related to control crest vertical curve rates, as adopted by AASHTO. The process revealed numerous SSD shortage areas, where the authors provided revised crest vertical curvature rates, in order to grant SSD adequacy throughout the vehicles' breaking process.

Furthermore, the authors aiming to provide the designers with ready-to-use vertical design tool associated the amended vertical curvature rates to AASHTO's road functional classification, as a function of the crest vertical curve's exit grade value.

INTRODUCTION

Sight distance is the length of roadway ahead that is visible to the driver (1). The minimum sight distance known as Stopping Sight Distance (SSD), is a highway geometric design element of fundamental importance. SSD must be provided at every point along the road surface, thus affecting critical road design parameters which directly impose economic considerations on both new road designs as well as road improvement projects (e.g. 1-4).

The American Association of State Transportation Officials (AASHTO) design guide titled “A Policy on Geometric Design of Highways and Streets” (commonly referred to as the Green Book) notes that for vertical curves, the grade effect is somewhat balanced and there is no need to adjust SSD due to grade (1). However, there is a significant difference in SSD values between upgrades and downgrades also noted in the Green Book (1), Moreover, the Green Book guidelines state that *the minimum lengths of crest vertical curves, based on sight distance criteria, generally are satisfactory from the standpoint of safety, comfort and appearance*, implying that the vertical curvature rate is adequately determined through the suggested maximum grade control values, at least regarding normal design cases and eliminating areas such as decision areas (e.g. ramp exit gores etc.).

Since maximum grade values vary depending on the road’s functional classification, the objective of the paper is to investigate the sufficiency of the suggested crest vertical curvature rates by AASHTO from the grade control point of view considering that their current definition is based on a level road surface.

BACKGROUND

According to existing design policies (e.g. 1-4), the SSD of a vehicle consists of two distance components: the distance traveled during driver’s perception – reaction time to the instant the brakes are applied and the distance while braking to stop the vehicle. For example, the SSD model adopted by the Green Book is represented by Equation (1).

$$SSD = V_o t_{pr} + \frac{V_o^2}{2g\left(\frac{a}{g} + s\right)} \qquad (1)$$

where :

V_o (m/sec) : vehicle initial speed

t_{pr} (sec) : driver’s perception – reaction time [2.5sec; AASHTO, 2011]

g (m/sec^2) : gravitational constant [9.81m/sec^2 (32.2ft/sec^2)]

a (m/sec^2) : vehicle deceleration rate [3.4m/sec^2 (11.2ft/sec^2); AASHTO, 2011]

s (%/100) : road grade [(+) upgrades, (-) downgrades]

Current road design standards (e.g. 1-4), determine minimum lengths of crest vertical curves as well as the consequent rate of vertical curvature based on SSD provision. Equation 2 and Equation 3 illustrate the parameters utilized in determining the length of crest vertical curves (L), where the vertical curvature rate definition (K) is shown in Equation 4. The values of K derived for SSD<L, apply without significant difference also for the case SSD>L.

$$L = \frac{(s_2 - s_1)SSD^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} \qquad SSD<L \qquad (2)$$

$$L = 2SSD - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{s_2 - s_1} \qquad SSD>L \qquad (3)$$

$$K = \frac{L}{s_2 - s_1} \qquad (4)$$

where:
K : vertical curvature rate (m)
L : length of vertical curve (m)
SSD : stopping sight distance (m)
h_1 : driver eye height (m) [1.08m (3.50ft); AASHTO 2011]
h_2 : object height (m) [0.60m (2.00ft); AASHTO 2011]
s_1, s_2 : grade values (%)

Most of the current efforts to evaluate SSD adequacy are based on 2-Dimensional models. Moreover, such efforts present a fragmented approach (i.e., examination of single elements) in investigating the adequacy of SSD and may underestimate or overestimate the available sight distance and thus possibly lead to safety violations (5).

The use of the vertical profile is a common approach in determining adequacy of SSD and typically roadway geometry is evaluated to ensure proper SSD requirements. This approach however fails to examine the continuity of the vertical alignment especially in crest curves and their exiting grades. This was noted in the past as a potential safety issue (6, 7).

There has been very little, if any, work on this topic even though there is the potential for requiring different lengths for vertical crest curves when exit grades are considered.

Table 1 illustrates the adopted design control values (rounded values) in the Green Book for SSD and crest vertical curvature rates respectively regarding various design speed values, based on the above equations (1). It should be noted here that these values are reflective of daylight conditions, since in most design policies the majority of guidance is provided for daylight conditions unless otherwise is developed (e.g., sag curves).

From Equations 2 and 4, it can be seen that the delivered crest vertical curvature rate is not grade dependent. The grade effect is indirectly introduced from the SSD determination, where regarding the AASHTO guidelines, 0 percent is assumed.

TABLE 1 Design Control Values for SSD and Crest Vertical Curvature Rates

Metric			US Customary		
V_{design} $^{(km/h)}$	SSD $^{(m)}$	K $^{(m)}$	V_{design} $^{(mph)}$	SSD $^{(ft)}$	K $^{(ft)}$
50	65	7	30	200	19
60	85	11	40	305	44
70	105	17	45	360	61
80	130	26	50	425	84
90	160	39	55	495	114
100	185	52	60	570	151
110	220	74	70	730	247
120	250	95	75	820	312
130	285	124	80	910	384

In other design guidelines, although the same equations are utilized, the adoption of control crest vertical curvature rates is addressed through various considerations.

For example in the German RAA design guidelines for freeways (2) the SSD values used for the crest vertical curvature rate determination are reached for most unfavorable (negative) grade values.

From another viewpoint, the relevant Greek OMOE-X design guidelines, in cases of two-lane rural roads, introduce a +10km/h safety margin in the SSD calculation. For example in order to determine the crest vertical curvature rate for a design speed of 60km/h, the SSD value utilized refers to 70km/h (3).

The possible deficiency of the current SSD determination approach as adopted in the Green Book will be furthermore examined by introducing the grade effect during braking on crest vertical transitions.

BRAKING CALCULATION ON VARIABLE GRADES

The current road design practices sufficiently address through Equation 1 the grade effect during the SSD procedure. However, the braking distance calculation for crest or sag curves that have variable grades is based on rather balanced assumptions. In most cases the algebraic mean grade value of the tangents preceding and succeeding the vertical curve is adopted. This concept though, fails to deliver the actual braking distances in all cases where the braking distance is less than the vertical curve length.

The work presented here regarding the evaluation of the effect of the variable grade during the braking process is based on a recently developed practice by the authors (7) briefly presented below.

Simple considerations based on the laws of mechanics through Equations 5 and 6 were applied, assuming time steps of 0.01sec, in order to determine both the instantaneous vehicle speed and pure braking distance (SSD minus distance travelled during driver's perception-reaction time).

$$V_{i+1} = V_i - g\left(\frac{a}{g} + s\right)t \qquad (5)$$

$$BD_i = V_i t - \frac{1}{2}g\left(\frac{a}{g} + s\right)t^2 \qquad (6)$$

where :
V_i (m/sec) : vehicle speed at a specific station i
V_{i+1} (m/sec) : vehicle speed reduced by the deceleration rate for t = 0.01sec
t (sec) : time fragment (t = 0.01sec)
s (%/100) : road grade in i position [(+) upgrades, (-) downgrades]
BD_i (m) : pure braking distance

By applying Equations 5 and 6 subsequently there is a sequence value $i=k-1$ where V_k becomes equal to zero. The corresponding value of ΣBD_{k-1} represents the total vehicle pure braking distance for the initial value of vehicle speed. The variable grade SSD is produced by adding the final pure braking distance to the distance travelled during the driver's perception – reaction time (first component of Equation 1) as follows:

$$SSD = V_o t_{pr} + \sum BD_{k-1} \qquad (7)$$

where :
V_o (m/sec) : vehicle initial speed
$\sum BD_{k-1}$ (m): total vehicle pure braking distance for the initial value of vehicle speed

Summarizing the SSD determination on variable grade values, the formula shown in Equation 1 is used, enriched by the actual grade value portions.

CREST VERTICAL CURVATURE RATE ADEQUACY INVESTIGATION

The potential Green Book inadequacy regarding the suggested vertical curvature rates, must be sought in the negative grade area since on one hand downgrades increase the SSD for a vehicle and on the other, the current vertical curvature rate definition is extracted assuming flat vertical geometry.

In the following paradigm, an investigation regarding the crest vertical curvature rate sufficiency assuming 70km/h (45 mph) design speed is carried out, by defining the actual SSD values along two specified positions. Figure 1 illustrates the length of the consequent vertical curve adopted by the Green Book (K=17m or K= 61ft), where the approach and exit grade values were set to +10% and -10% respectively. Two cases of vehicle braking are shown:

- Case 1, where the braking procedure begins at the starting point of the vertical curve (s=10%) where SSD=96.3m (316ft)
- Case 2, where the braking procedure begins at the midpoint of the vertical curve (s=0%) where SSD=113.0m (370ft)

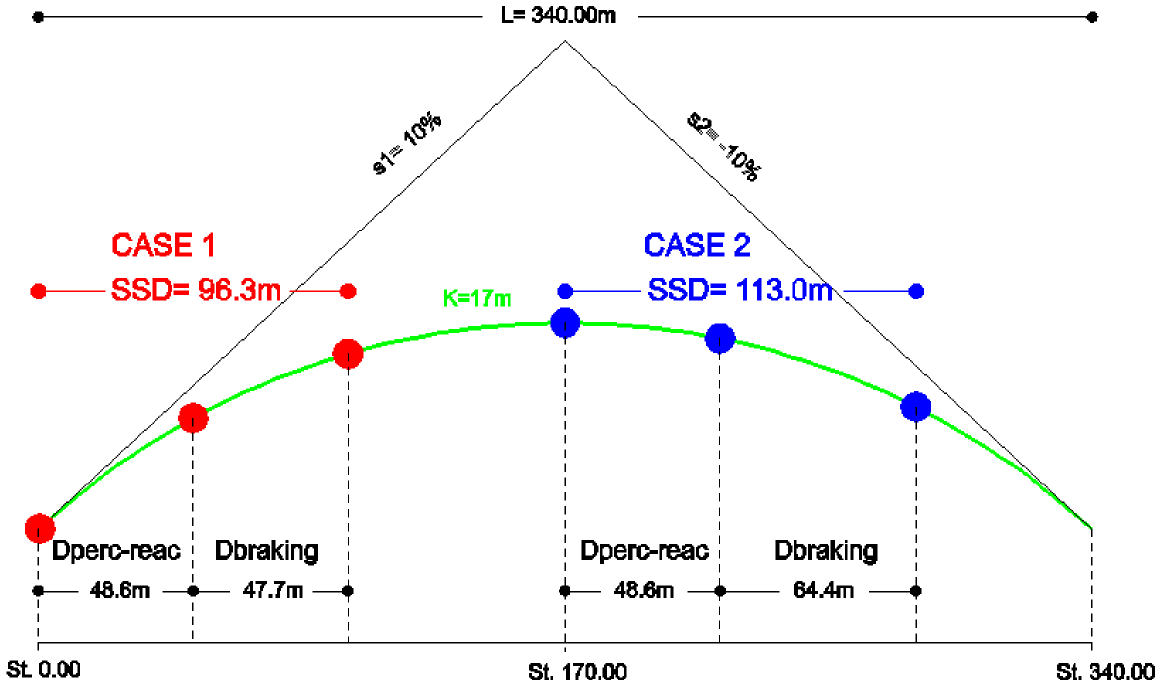
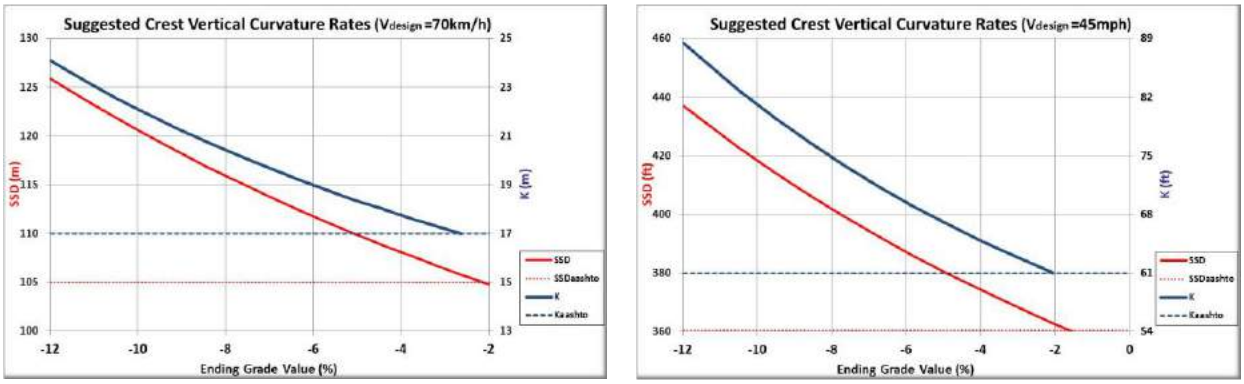


FIGURE 1 Discriminating Cases of SSD Variation on Crest Vertical Curvature Rate Suggested by AASHTO [V_{design}=70km/h (45mph), K=17m (61ft)].

In both cases the SSD definition was based on the calculation procedure described previously. As expected, the SSD value in Case 2 is greater not only from the relevant SSD value in Case 1, but from the SSD Design Control of 105m (360ft) in Table 1 for the 70km/h (45mph) design speed as well. This indicates that there are areas along the vertical curve t where the braking procedure requires greater SSD values and thus an increase of the crest vertical curvature rate in these cases seems indispensable.

Figure 2a illustrates certain SSD values extracted for vehicle braking under 70km/h (design speed) and the corresponding K value (K=17m). The relevant values drawn for the equivalent speed (approximately) in US Customary units are shown in Figure 2b. The SSD values of Figure 2 were drawn arranging the total braking procedure to be performed entirely inside the crest vertical curve. The horizontal axis of Figure 2 exhibits the grade value within the crest vertical curve where the vehicle is supposed to immobilize and is referred as ending grade value.

The data shows that as the downgrade becomes steeper, the required SSDs increase accordingly as well. In cases where the required SSDs exceed the relevant values suggested in the Green Book, the suggested K values are introduced in the secondary vertical axis. For example, in Figure 2b when an ending grade of a crest vertical curve is set to -8% and the length of the variable grade area is above400ft, in order to grant SSD adequacy, the minimum crest vertical curvature rate value must be approximately K=75ft, as opposed to the currently suggested K=61ft.

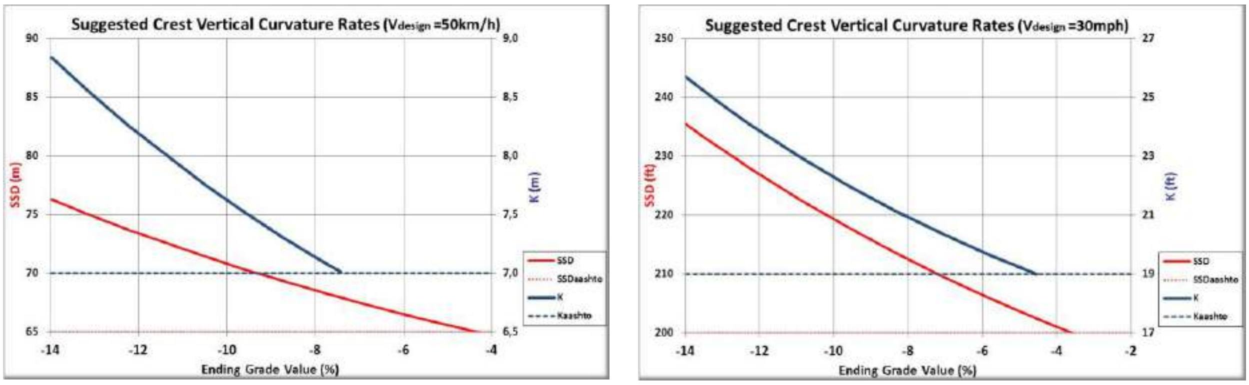


(a) V_{design}=70km/h (b) V_{design}=45mph

FIGURE 2 Suggested Crest Vertical Curvature Rate Values Based on SSD Adequacy.

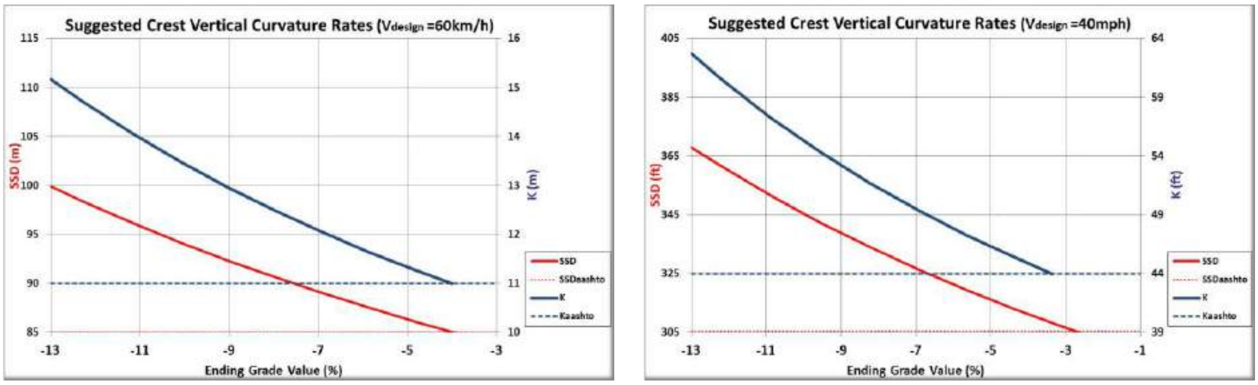
In order the above procedure to be more integrated, Figure 3 through Figure 10 illustrate the suggested crest vertical curvature rates based on the vertical curve's ending grade value for design speed values of 50km/h, 60km/h, 80km/h, 90km/h, 100km/h, 110km/h, 120km/h and 130km/h, paired with the equivalent US Customary units respectively. These figures assess the braking effect on steep (mostly) variable downgrades and thus deliver ready-to-use crest vertical curvature rate values for designers.

At first glance of Figures 2 to 10, one would expect a similar variation between the values outlined through Case (a) and Case (b). However, even for exactly equivalent speed values, random rounding concepts result in slight variation on the delivered SSD and K values respectively.



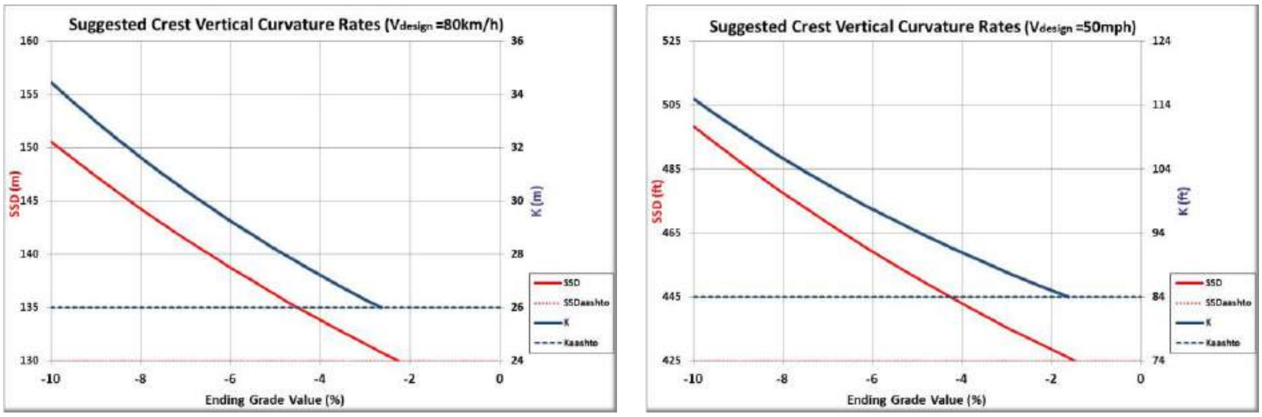
(a) V_{design}=50km/h (b) V_{design}=30mph

FIGURE 3 Suggested Crest Vertical Curvature Rate Values Based on SSD Adequacy.



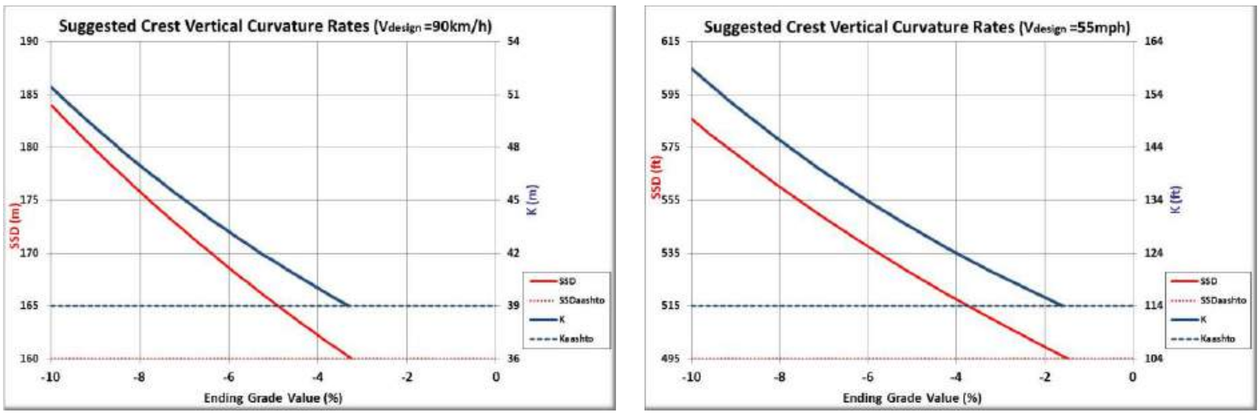
(a) V_{design}=60km/h (b) V_{design}=40mph

FIGURE 4 Suggested Crest Vertical Curvature Rate Values Based on SSD Adequacy.



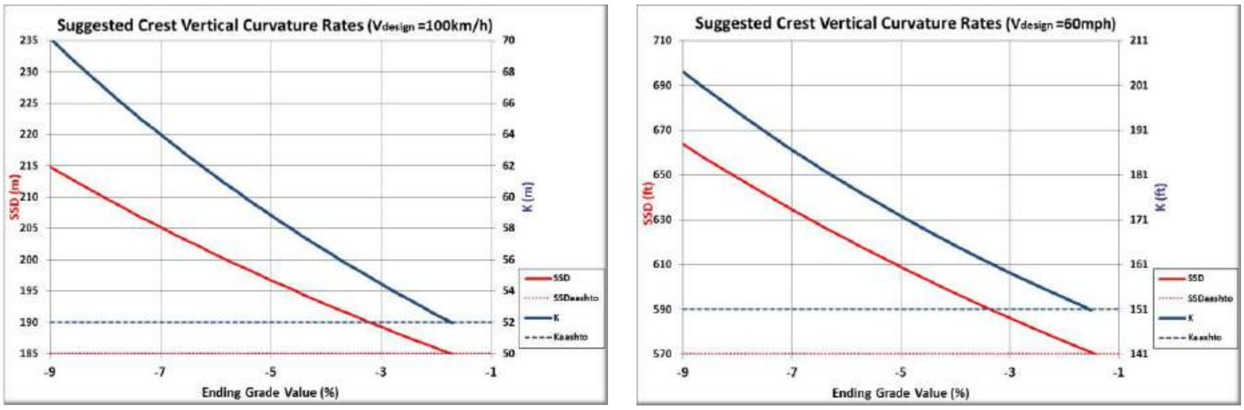
(a) V_{design}=80km/h (b) V_{design}=50mph

FIGURE 5 Suggested Crest Vertical Curvature Rate Values Based on SSD Adequacy.



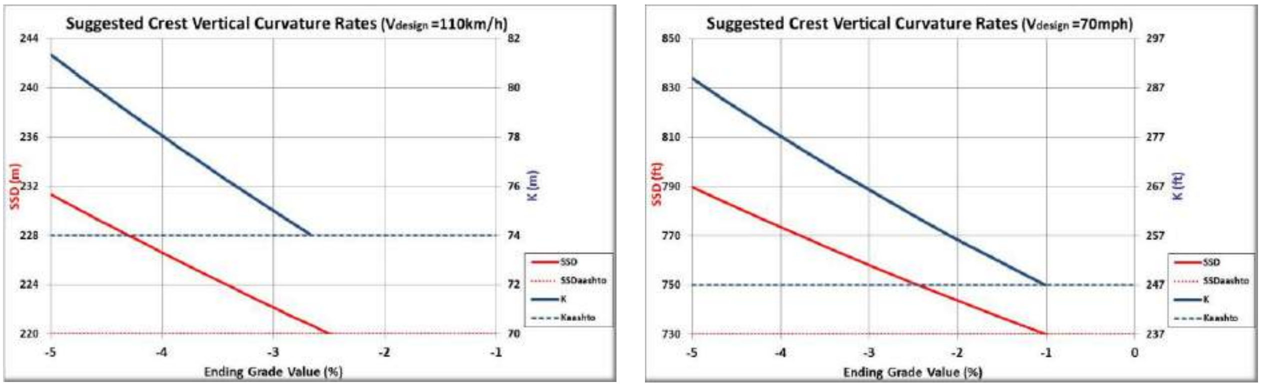
(a) V_{design}=90km/h (b) V_{design}=55mph

FIGURE 6 Suggested Crest Vertical Curvature Rate Values Based on SSD Adequacy.



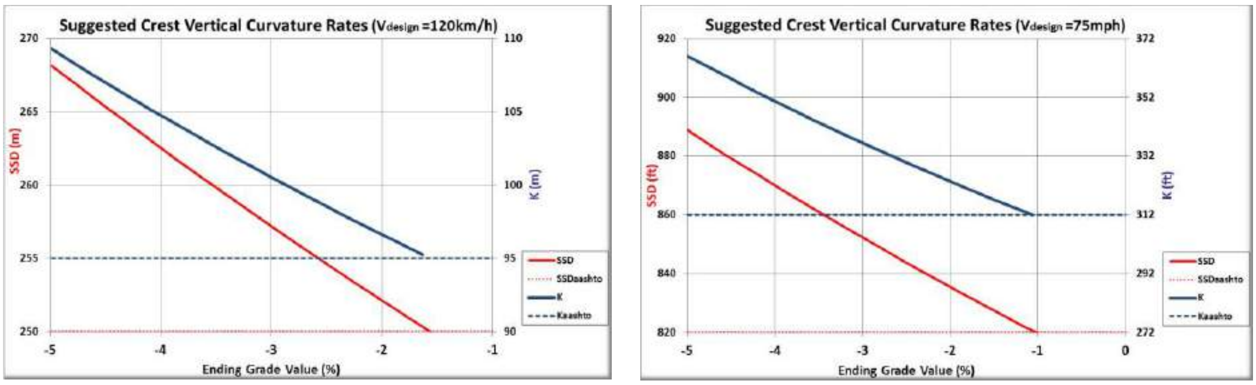
(a) V_{design}=100km/h (b) V_{design}=60mph

FIGURE 7 Suggested Crest Vertical Curvature Rate Values Based on SSD Adequacy.



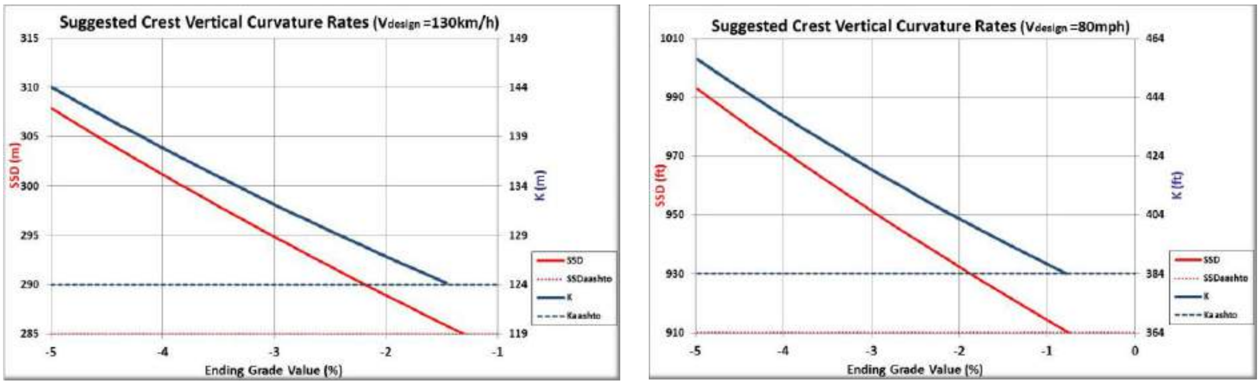
(a) V_{design}=110km/h (b) V_{design}=70mph

FIGURE 8 Suggested Crest Vertical Curvature Rate Values Based on SSD Adequacy.



(a) V_{design}=120km/h (b) V_{design}=75mph

FIGURE 9 Suggested Crest Vertical Curvature Rate Values Based on SSD Adequacy.



(a) V_{design}=130km/h (b) V_{design}=80mph

FIGURE 10 Suggested Crest Vertical Curvature Rate Values Based on SSD Adequacy.

The ending grades utilized in Figures 2 through 10, were drawn based on grade control criteria as found in AASHTO's road functional classification. For example in Figure 3, the selection of grade values up to 14% for 50km/h (80mph) design speed refers to mountainous local rural roads as recommended in the Green Book.

Figures 2-10 show that there is a horizontal variation between the SSDs' intersection point (red line intersection) and the relevant K values intersection point (blue line intersection). Theoretically these two intersection points should coincide horizontally, since both refer to the same ending grade value. However, this is not the case, since, the crest vertical curvature rates calculation formula based on Equations 2 and 4, deliver values which are rounded for design. For example assuming two extreme cases:

- V_{design}=50km/h, the K value is calculated 6.4m but introduced as 7.0m in (1)
- V_{design}=90km/h, the K value is calculated 38.9m but introduced as 39.0m in (1)

In the first case, the adopted K value for design is increased by 9.4%, where in the second only 0.3%. This explains the horizontal variation between the above stated intersection points.

CORRELATION BETWEEN SUGGESTED CREST VERTICAL CURVATURE RATES VALUES AND ROAD FUNCTIONAL CLASSIFICATION

The above analysis revealed a SSD inadequacy on the downgrade area of steep grades when control crest vertical curvature rates based on AASHTO design guidelines are utilized. On the other hand, as each road class is associated to a particular grade range, its impact in the above process is ignored.

The correlation between AASHTO's road functional classification and the amended crest vertical curvature rates as calculated here is shown in Table 2. It should be noted that this is illustrated for a single design speed of 80km/h (50mph) for all possible functional classification categories.

Table 2, illustrates an example for ready-to-use values of crest vertical curvature rates, based on the desired exiting grade value of the design. The illustrated crest curvature rates refer to negative exit grade values, extracted below a certain breakpoint where SSD inadequacy commences as shown in Figure 5, and can be applied in cases where the length of the vertical curve exceeds the required SSD values as shown.

The gaps in the suggested crest vertical curvature rates indicate that the maximum grade value of the certain road class is lower.

TABLE 2 Suggested Crest Vertical Curvature Rate Values Based on Roadway's Functional Classification for V_{design}=80km/h (V_{design}=50mph)

Note: L refers to length of vertical curve; values are shown in both **Metric (m)** and **US Customary Units (ft)**

<table>
<tr><th rowspan="2"></th><th rowspan="2">Type of Terrain</th><th rowspan="2">AASHTO</th><th colspan="8">Exit Grade Value (%)</th></tr>
<tr><th>-3</th><th>-4</th><th>-5</th><th>-6</th><th>-7</th><th>-8</th><th>-9</th><th>-10</th></tr>
<tr><td rowspan="3">Local Rural</td><td>Level</td><td rowspan="3">26m
84ft</td><td rowspan="3">27m
(L>132m)
88ft
(L>436ft)</td><td rowspan="3">28m
(L>134m)
91ft
(L>443ft)</td><td rowspan="3">29m
(L>136m)
95ft
(L>451ft)</td><td rowspan="3">30m
(L>139m)
98ft
(L>459ft)</td><td>-</td><td>-</td><td rowspan="2">-</td><td rowspan="2">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">31m
(L>142m)
102ft
(L>468ft)</td><td rowspan="2">32m
(L>144m)
106ft
(L>478ft)</td></tr>
<tr><td>Mountainous</td><td>33m
(L>147m)
111ft
(L>488ft)</td><td>35m
(L>151m)
115ft
(L>498ft)</td></tr>
<tr><td rowspan="3">Rural Collectors</td><td>Level</td><td rowspan="3">26m
84ft</td><td rowspan="3">27m
(L>132m)
88ft
(L>436ft)</td><td rowspan="3">28m
(L>134m)
91ft
(L>443ft)</td><td rowspan="3">29m
(L>136m)
95ft
(L>451ft)</td><td rowspan="3">30m
(L>139m)
98ft
(L>459ft)</td><td>-</td><td rowspan="2">-</td><td rowspan="2">-</td><td rowspan="3">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">31m
(L>142m)
102ft
(L>468ft)</td></tr>
<tr><td>Mountainous</td><td>32m
(L>144m)
106ft
(L>478ft)</td><td>33m
(L>147m)
111ft
(L>488ft)</td></tr>
<tr><td rowspan="3">Rural Arterials</td><td>Level</td><td rowspan="3">26m
84ft</td><td rowspan="3">27m
(L>132m)
88ft
(L>436ft)</td><td rowspan="3">28m
(L>134m)
91ft
(L>443ft)</td><td>-</td><td>-</td><td>-</td><td rowspan="3">-</td><td rowspan="3">-</td><td rowspan="3">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">29m
(L>136m)
95ft
(L>451ft)</td><td>-</td><td>-</td></tr>
<tr><td>Mountainous</td><td>30m
(L>139m)
98ft
(L>459ft)</td><td>31m
(L>142m)
102ft
(L>468ft)</td></tr>
<tr><td rowspan="3">Urban Collectors</td><td>Level</td><td rowspan="3">26m
84ft</td><td rowspan="3">27m
(L>132m)
88ft
(L>436ft)</td><td rowspan="3">28m
(L>134m)
91ft
(L>443ft)</td><td rowspan="3">29m
(L>136m)
95ft
(L>451ft)</td><td rowspan="3">30m
(L>139m)
98ft
(L>459ft)</td><td rowspan="3">31m
(L>142m)
102ft
(L>468ft)</td><td>-</td><td rowspan="2">-</td><td rowspan="2">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">32m
(L>144m)
106ft
(L>478ft)</td></tr>
<tr><td>Mountainous</td><td>33m
(L>147m)
111ft
(L>488ft)</td><td>35m
(L>151m)
115ft
(L>498ft)</td></tr>
<tr><td rowspan="3">Urban Arterials</td><td>Level</td><td rowspan="3">26m
84ft</td><td rowspan="3">27m
(L>132m)
88ft
(L>436ft)</td><td rowspan="3">28m
(L>134m)
91ft
(L>443ft)</td><td rowspan="3">29m
(L>136m)
95ft
(L>451ft)</td><td rowspan="3">30m
(L>139m)
98ft
(L>459ft)</td><td>-</td><td rowspan="2">-</td><td rowspan="2">-</td><td rowspan="3">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">31m
(L>142m)
102ft
(L>468ft)</td></tr>
<tr><td>Mountainous</td><td>32m
(L>144m)
106ft
(L>478ft)</td><td>33m
(L>147m)
111ft
(L>488ft)</td></tr>
<tr><td rowspan="3">Freeways</td><td>Level</td><td rowspan="3">26m
84ft</td><td rowspan="3">27m
(L>132m)
88ft
(L>436ft)</td><td rowspan="3">28m
(L>134m)
91ft
(L>443ft)</td><td>-</td><td rowspan="2">-</td><td rowspan="3">-</td><td rowspan="3">-</td><td rowspan="3">-</td><td rowspan="3">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">29m
(L>136m)
95ft
(L>451ft)</td></tr>
<tr><td>Mountainous</td><td>30m
(L>139m)
98ft
(L>459ft)</td></tr>
</table>

CONCLUSIONS

The paper investigates the consequence of the Green Book guidelines, to adopt minimum crest vertical curvature rates based on leveled grade values.

Initially, the authors addressed the SSD calculation on variable grades during the braking process through an earlier approach based on the point mass model and the laws of mechanics. This process resulted in determining that the negative grade area of crest vertical curves, as expected, increases the calculated SSD values.

Subsequently, for a wide range of design speed values, charts of the required SSDs were drawn as a function of various ending grade values, based on control crest vertical curve rates, as adopted by AASHTO. Since various SSD shortage areas appeared in the charts, the authors provided amended crest vertical curvature rates as well, in order to grant SSD adequacy throughout the braking process. The resultant crest vertical rates apply for cases where the length of the vertical curve exceeds the calculated SSD values.

An immediate implementation of the present approach is to provide the designers with ready-to-use revised crest vertical curvature rates, based on the desired exiting grade value of the design in accordance to roadway's functional classification as adopted by AASHTO.

However further analysis is required in order to include the effect of combined horizontal – vertical alignment, certain arrangements of which might impose additional restrictions.

Moreover, additional qualitative research seems necessary to evaluate parameters of SSD (braking on curves, ABS braking, friction coefficient etc.) as well, in order to reflect current vehicle dynamics trends and thus simulate the braking procedure more realistically. One should not ignore the fact that the human factor might impose additional restrictions and consequently influence the braking process to some extent beyond the perception-reaction procedure and friction reserve utilized in the braking process.

Finally, it is also necessary to underline the fact that the parameters used in the present paper (speed values, perception reaction time etc.) refer to daylight driving conditions, as the vehicle speed values in night time driving conditions are 6km/h – 15km/h less (8) on one hand and on the other the road view geometry changes.

REFERENCES

1. American Association of State Highway and Transportation Officials (AASHTO). *A Policy on Geometric Design of Highways and Streets*, Fifth Edition. Washington, DC., 2011
2. Ed.German Road and Transportation Research Association, Committee, Geometric Design Standards. *Guidelines for the Design of Freeways, (RAA)*, Germany, 2008.
3. Ministry of Environment, Regional Planning and Public Works. *Guidelines for the Design of Road Projects, Part 3, Alignment (OMOE-X),* Greece, 2001.
4. Ministerio de Fomento. *Instrucción de Carreteras, Norma 3.1 – IC "Trazado",* Spain, 2000.
5. Hassan, Y., Easa, S.M. and Abd El Halim A.O. Design Considerations for Combined Highway alignments. *Journal of Transportation Engineering,* Vol 123, No.1, pp. 60-68, 1997.
6. Moreno Chou, A., Perez, V., Garcia, A. and Rojas M. Optimal 3-D Coordination to Maximize the Available Stopping Sight Distance in Two – Lane Roads. *Proceedings of the 4th International Symposium on Highway Geometric Design,* TRB, Valencia Spain, 2010.
7. Mavromatis S., Palaskas S. and Psarianos B. Continuous Three-Dimensional Stopping Sight Distance Control on Crest Vertical Curves. Paper published *on the Advances in Transportation Studies (ATS),* XXVIII issue, 2012, pp.17-30.
8. Malakatas, K. *Operating Speed Predicting Model On Two-Lane Rural Roads During Night-Time.* Diploma Thesis, National Technical University of Athens, 2012.