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# Exploring Crash Injury Severity on Urban Motorways by Applying Finite Mixture Models

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## Abstract

The effective treatment of crashes is a major concern to societies due to losses in human lives and the economic and social costs. Crash injury severity is one of the most well-known and well researched aspects of crashes in the science of road safety, with many study types across different countries and sample frames. All types of severity analyses primarily aim to determine the contributing factors for increased crash or injury severity and the respective degree of said contributions. Usually, driver and passenger-related factors are examined, as well as road environment characteristics such as geometric design aspects, traffic conditions, vehicle type and technology and weather conditions, as investigated in past literature. However, crash injury severity has not been adequately explored as the majority of relative research has a focus on analyzing crash likelihood, i.e. the probability of a crash to occur. Thus, this study aims to add to current knowledge by examining the determinants behind injury severity of occupants involved in crashes in the urban motorway Attica Tollway ('Attiki odos') which lies in the Greater Athens Area in Greece. In order to account or the unobserved heterogeneity, crash injury severity is explored by utilizing finite mixture logit models. Results indicate that a number of traffic parameters such as truck proportion, average flow and standard deviation of occupancy, as well as other risk factors, such as accident type and engine size have a significant effect on the injury severity outcome of vehicle occupants. Moreover, this model accounted for the heterogeneity among two distinct groups of observations. More research is needed in order to incorporate also vulnerable road users such as motorcyclists, cyclists and pedestrians and further investigate the impact of real-time traffic and weather parameters.

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*Keywords:* Urban motorway; crash severity; real-time data; finite mixture models

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## 1. Introduction

The effective treatment of crashes is a major concern to societies due to losses in human lives and the economic and social costs. Crash injury severity is one of the most well-known and well-researched aspects in the science of

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road safety, with many study types across different countries and sample frames. In the following an indicative snapshot of the literature from several noteworthy studies is provided, however the list is not exhaustive. All types of severity analyses primarily aim to determine the contributing factors for increased crash or injury severity, and the respective degree of said contributions. Usually, driver and passenger-related factors are examined, as well as road environment characteristics such as geometric design aspects, traffic conditions, vehicle type and technology and weather conditions, as investigated in several relevant studies (Al-Ghamdi 2002; Quddus et al. 2002; Yamamoto and Shankar 2004; Yau 2004; Chang and Wang 2006; Savolainen et al., 2011; Milton et al. 2008). Injury severity is usually examined as a categorical variable, expressed by two or more discrete categories according to the result of the crash (for instance property damage only, slightly injured, severely injured, and killed).

Certain studies have investigated safety effects on crash severity using real-time traffic data. Golob et al. (2008) have utilized single-loop detector data to develop a tool used for real-time monitoring of the safety level of freeways, or for forecasting the safety of changes in traffic flows. They avoided utilizing speed or density directly, due to mathematical and technical constraints, preferring a set of eight traffic factors related to crashes instead. The models developed alerted transport managers to conditions detrimental to road safety or provided more general guidance towards tendencies of potentially dangerous conditions.

Quddus et al. (2010) proposed using disaggregated crash data to investigate the influence of the level of traffic congestion and crash severity, via utilizing various ordered response models. Data were collected from a UK orbital motorway and several other characteristics were controlled for, traffic characteristics (e.g. traffic flow) being amongst them. Traffic congestion was not found to affect crash severity for the motorway, while increases in traffic flow was found to reduce the severity of crashes when analyzed with disaggregated data, contrary to the existing literature up to that point. Yu and Abdel-Aty (2014a, 2014b) found that the standard deviation of speed increases crash severity. Yu and Abdel-Aty (2014a) applied Hierarchical Bayesian probit models and found that large speed variations and low visibility increase crash severity. Christoforou et al. (2010) investigated occupant injury severity on a specific junction in Paris and found that increased traffic volume is associated with less severe injuries.

Another study undertook a network approach that aggregated pre-crash traffic and geometric data (Imprialou et al., 2016) in order to avoid over aggregation of traffic variables and the corresponding information losses, integrating Bayesian multivariate Poisson lognormal regression for modelling purposes. Speed was found to be statistically significant for crash occurrence and severity with the condition-based approach, while for the link-based model speed has a negative relationship with crash occurrences for all severity types. The authors selected the condition-based approach due to its basis on crash time and location as more representative of real circumstances.

Regarding weather effects, there have been a number of studies investigating the impact of rainfall on crash severity, but the overall findings have been contradictory. At times a positive correlation is reported, (Caliendo et al., 2007), occasionally, a negative relationship is also reported (Theofilatos et al., 2012). Other weather factors that can contribute to increased crash severity include fog and similarly visual-impairing conditions (Al-Ghamdi, 2007; Abdel-Aty et al., 2011). Yu and Abdel-Aty (2014a), found that low visibility is a critical contributory factor in crash severity outcomes in snow seasons. However, another study by the same authors (Yu and Abdel-Aty, 2014b) indicates that the snow season is associated with slight injury crashes. Yu and Abdel-Aty (2014b) suggested that higher temperatures are correlated with lower severity outcomes.

Moreover, a study investigated crash injury severities using switching multinomial logit models in order to capture unobserved heterogeneity (Malyshkina and Mannering, 2009). Adverse weather conditions did not increase crash severity in a statistically significant manner. It is proposed that adverse weather increases the number of serious and non-serious crashes at about the same rate, so the overall proportions remains approximately the same. This is perhaps attributed to the increase of caution on the drivers' part in adverse weather conditions. Interestingly, when comparing those findings to previous literature finds (Malyshkina et al., 2009), the authors conclude that it is possible that those compensation effects mitigate adverse weather effects enough to prevent more severe injuries, while crash numbers may increase in absolute numbers.

Overall, real-time traffic data combined with weather data are rarely utilized to create models investigating crash severity, however it was done so in the following studies. Jung et al. (2010) examined single-vehicle crashes on Wisconsin interstate highways utilizing polychotomous response models. They found that, amongst other variables, water film depth, temperature, stopping sight distance were statistically significant for crash severity, however wind speed and direction was found to lead to less severe crashes. Similarly, Yu and Abdel-Aty (2014a and 2014b)

investigated crash injury severity on for a mountainous freeway and an urban expressway, and concluded that real-time traffic and weather variables have substantial influences on crash injury severity, and potential forecasting and prevention uses.

Recently, real-time traffic and weather data are utilized when analyzing road safety in freeways in order to develop proactive safety management systems. However, crash injury severity has not been adequately explored as the majority of relative research has a focus on analyzing crash likelihood, i.e. the probability of a crash to occur. Thus, this study aims to add to current knowledge by examining the determinants behind injury severity of occupants involved in crashes in the urban motorway Attica Tollway (‘Attiki odos’) which lies in the Greater Athens Area in Greece. In order to account or the unobserved heterogeneity, crash injury severity is explored by utilizing finite mixture logit models. More specifically, injury severity is defined as a binary dichotomous variable, namely killed/severely injuries and slight injuries (KSI vs SI).

## 2. Data preparation

The crash database consists of 387 injury cases which occurred between 2006 and 2011. The required crash data for Attica Tollway was extracted from the Greek crash database SANTRA provided by the Department of Transportation Planning and Engineering of the National Technical University of Athens. The unit of analysis was any vehicle occupant involved in a crash (rider, driver or passenger) resulting in at least one person being slightly injured. Therefore, each one of the 387 injury cases in the dataset is a record of the severity level sustained by each vehicle occupant involved in the crash. Therefore, a single crash corresponds to various observations that are equal to the number of all injured persons involved in this crash. 344 persons were slightly injured, while 43 were severely injured or killed.

Table 1. Summary statistics of crash related variables.

Variable	Type	Description	Descriptive statistics	
Severity	Dummy	Killed/Severely injured=0	344	88.89%
		Severe=1	43	11.11%
Vehicle type	Dummy	Car	205	52.97%
		PTW	161	41.60%
		Other	21	5.43%
Illumination	Dummy	Day=1	283	73.13%
		Night/Dusk=0	104	26.87%
		Off road/Fixed object/Other	116	29.97%
Accident Type (collision type)	Dummy	Rear-end	157	40.57%
		Side	43	11.11%
		Sideswipe	71	18.35%
Engine size	Continuous	Numeric in cc	mean=1167.553	st.dev= 1303.694
Age of occupant	Continuous	Numeric in years	mean=37.7804	st.dev=14.648
Gender	Dummy	Male=1	283	73.13%
		Female=0	104	26.87%
Nationality of occupant	Dummy	Greek=1	357	92.25%
		Other=0	30	7.75%
Road curvature	Dummy	Straight line=1	313	80.88%
		Curve=0	74	19.12%

To achieve the aims of the study, real-time traffic and weather parameters were considered and processed. Regarding traffic parameters, a 1-hour time series of traffic flow, occupancy, speed and truck proportion (in 5-

minutes intervals ending 5 minutes before the time of the crash) were extracted for each crash. This time lag was used in order to avoid the impact of the crash itself on the traffic variables and also to compensate for any ‘inaccuracies’ in the exact time of the crash. For example, if a crash occurred at 20:00, the traffic data considered were obtained from the 18:55-19:55 time period. Similar techniques have been applied in other past real-time data papers. The raw 5-min traffic data before the time of the crash occurrence (considering the 5-min time lag that was introduced), were extracted from the closest upstream loop detector and then were further aggregated in 15min, 20min, 30min and 60min in order to obtain maxima, averages, standard deviations, medians and coefficients of variation. In cases when loop detectors suffered from problems that might have resulted in unreasonable values for speed, volume, and occupancy. Such values (e.g. occupancy>100%, speed>200 km/h or speed>0 along with flow=0) were discarded from the database. Crashes with traffic data unavailability were also discarded.

Concerning the weather data, the weather records for each meteorological station covered the whole period from 2006 to 2011. For that reason, each crash case had to be assigned to the closest meteorological station and then the relevant weather data had to be extracted. The 10-min raw data were aggregated in order to obtain maxima, averages and standard deviations, for 1-hour prior to the time of the crash occurrence (considering the 5-min time lag). Regarding rainfall, the total amount of rain has also been calculated for 1, 2, 6 and 12 hours prior to the time of the crash after considering the 5min time-lag that was described earlier.

### 3. Methodology

Severity is explored through finite mixture logit models. These models are capable of accounting for unobserved heterogeneity, which has been recognized as a critical issue when analyzing accident severity (Savolainen et al., 2011). This happens because some of the factors affecting severity are not observable. Mannering and Bhat (2014), correctly argue, that when analyzing motorcycle severity, heterogeneity is major issue as it may occur from various unobserved factors (motorcycle speed, riders’ physical condition etc.), and consequently resulting in biased estimates.

Crash severity is explored through finite mixture logit models which can account for the unobserved heterogeneity by allowing parameters to vary across observations. More specifically, the purpose of the finite mixture logit model is to divide the sample into homogenous groups (latent classes). As a result, the finite mixture logit analysis is also referred as Latent Class analysis. Consequently, in finite mixture formulation parameter heterogeneity across observations is explored with a discrete distribution or set of classes. It is noted that the finite mixture model does not require to assume a distribution relating the way in which parameters vary across observations. The Finite Mixture modelling approach can be considered as an extension of the standard binary logit model based on a finite mixture approach in which the unobserved heterogeneity is accounted for via latent classes. In general, it includes a latent class model that captures the effect of unobserved variables on the binary outcome variable. Because of the fact that finite mixture analysis divides the sample into distinct classes with homogenous attributes, an important issue is the determination of the number of classes. For that reason, international literature indicates to implement the Bayesian information criterion (BIC) to overcome this issue. One disadvantage of the finite mixture model could be considered the fact that variation within observations in the same class are not allowed. In other words, unobserved heterogeneity in the each latent class itself cannot be accounted for, under this modeling formulation.

This logit modelling approach is an extension of the binary logit model based on a finite mixture approach in which the unobserved heterogeneity is accounted for via latent classes. The finite mixture logit analysis is also referred as Latent Class analysis.

The Finite Mixture Logit (FML) model can be considered as an extension of the standard binary logit model which also includes a latent class model that captures the effect of unobserved variables on the binary outcome variable. The response variable is  $Y$  and takes the values  $y = 0$  (non-event) and  $y = 1$  (event). Holm et al. (2009) suggest the FML model with  $J$  ( $j = 1, \dots, J$ ) latent classes as:

$$P(Y = 1|x) = \sum_{j=1}^J P(Y = 1|x, \varepsilon = \varepsilon_j) P(\varepsilon = \varepsilon_j) = \sum_{j=1}^J \frac{\exp(\alpha + \beta x + \varepsilon_j) P(\varepsilon = \varepsilon_j)}{1 + \exp(\alpha + \beta x + \varepsilon_j)} \quad (1)$$

where  $\alpha$  is a constant term,  $x$  is a vector of independent variables,  $\beta$  is a corresponding row vector of regression coefficients,  $\varepsilon_j$  is the effect of the  $j$ 'th latent class on the probability of observing  $Y = 1$ , and  $P(\Xi = \varepsilon_j)$  is the proportion of the population that belongs to the  $j$ 'th latent class.

According to Holm et al. (2009), the log-likelihood function  $\ln L$  for  $n$  independent observations can be defined as:

$$\ln L = \sum_{i=1}^n \ln P(Y = 1 | x_i) + (1 - y_i) \ln(1 - P(Y = 1 | x_i)) \quad (2)$$

$$P(Y = 1 | x_i) = p P_{0i} + (1 - p) P_{ie}, \quad (3)$$

$$P_{0i} = \frac{\exp(\alpha + \beta x_i)}{1 + \exp(\alpha + \beta x_i)} \quad (4)$$

$$P_{ie} = \frac{\exp(\alpha + \beta x_i + \varepsilon)}{1 + \exp(\alpha + \beta x_i + \varepsilon)} \quad (5)$$

where  $P(\Xi = 0) = p$  and  $P(\Xi = e) = 1 - p$ .

In finite mixture formulation, parameter heterogeneity across observations is explored with a discrete distribution or set of classes (Greene and Hensher, 2003; Shaheed and Gkritza, 2014). Finite mixture analysis divides the sample into distinct classes with homogenous attributes. As Greene and Hensher, (2003) correctly state, an important issue is the determination of the number of classes. It is also suggested the implementation of the Bayesian information criterion (BIC) where  $\ln L$  is the log-likelihood for the sample:

$$BIC(model) = \ln L + \frac{(\text{model size}) \ln N}{N} \quad (6)$$

#### 4. Results

The finite mixture analysis revealed two distinct classes of injured occupants with homogenous attributes; latent class 1 with probability 84.4% and latent class 2 with probability 15.6%. These are the probabilities that each person belongs in each latent class respectively. The optimum number of latent classes was determined according to the BIC criterion, with lower BIC indicating best models.

Table 2. Summary of the finite mixture logit model for occupant injury severity.

Variables	Latent Class 1			Latent Class 2		
	Mean	t-statistic	p-value	Mean	t-statistic	p-value
Constant term (random)	-1.720	-0.961	0.337	36.153	2.573	0.010
Acc.type0 (reference cat.)	-	-	-	-	-	-
Acc.type1 (fixed)	-2.338	-1.352	0.176	-2.338	-1.352	0.176
Acc.type2 (fixed)	-26.779	-2.262	0.024	-26.779	-2.262	0.024
Acc.type3 (fixed)	-7.196	-2.429	0.024	-7.196	-2.429	0.024
CC (fixed)	-0.002	-1.755	0.079	-0.002	-1.755	0.079
Tr.Prop_avg_30m_up (fixed)	-1.038	-2.429	0.015	-1.038	-2.429	0.015
Q_avg_30m_up (random)	0.027	1.362	0.173	-0.236	-2.354	0.019
Occ_stdev_30m_up (random)	154.060	2.597	0.009	-1629	-2.134	0.033
Log-likelihood at zero			-134.998			
Final Log-likelihood			-112.152			
Likelihood ratio test			45.692			
McFadden R <sup>2</sup>			0.169			

Table 2 summarizes the findings of the finite mixture logit model. The likelihood of the final model was -112.152 and the McFadden R-square was considered adequate with a value of 0.169, since it is suggested that values

between 0.2 and 0.4 are of very good fit. Moreover, the likelihood ratio test between the null and the full model is considered significant. Five explanatory variables were found to be significant (accident type, 30min average flow, 30min standard deviation of occupancy, 30min average truck proportion and engine size). The variables of the final model were checked for potential multicollinearity and after having ensured that they were not correlated, it was decided to be retained in the final model. Accident type, average truck proportion and engine size (CC) were set as fixed across latent classes, while the constant, the average flow and the standard deviation of occupancy were found to be random and free to vary across the two latent classes.

Consequently, elasticity analysis (marginal effects) is suggested in order to gain a better understanding of the effect of variables that differ across latent classes (Shaheed and Gritza, 2014; Xie et al., 2014; Cerwik et al., 2014). The elasticities and the pseudo-elasticities for each variable are presented on Table 3.

Table 3: Average elasticity and pseudo-elasticity of each variable for the finite mixture model for occupant injury severity.

Variable	e
Acc.type2	-0.843
Acc.type3	-0.933
CC	-0.558
Tr.Prop_avg_30m_up	-0.639
Q_avg_30m_up	-0.059
Occ_stdev_30m_up	0.086

It is observed that the type of accident was fixed across the two classes. It is shown, that side (Acc.type2) and sideswipe collisions (Acc.type3) were found to be statistically significant, while rear-end collisions (Acc.type1) was not significant. The negative signs of the beta coefficients indicate lower injury severities for the aforementioned significant variables than the reference category, which is collision with fixed object/run-off road (Acc.type0). The negative effect of these variables is consistent with Al-Ghamdi (2002) and Theofilatos et al. (2012). However, in Theofilatos et al. (2012), rear-end collisions were found to have a negative effect on accident severity outside urban areas.

The engine size (CC) has a negative sign across latent classes, indicating that occupants of vehicles with small engine are more likely to sustain severe injuries, regardless of the class that they belong, implying a consistent fixed and homogenous effect of engine size. This negative effect may be attributed to the fact, that heavy vehicles such as SUVs and trucks have large engine size offer more security to their occupants.

It is observed that average truck proportion (Tr.Prop\_avg\_30m\_up) has a fixed negative sign across latent classes. This means that when the proportion of heavy vehicles and trucks is lower, occupants are more likely to be severely or fatally injured. This is probably the first time that this variable is examined, when analyzing severity with real-time traffic data, and as such the understanding of the impact on occupant injury severity needs further investigation. In order to understand this effect, the effect of traffic flow needs to be co-examined. Average traffic flow (Q\_avg\_30m\_up) has a negative sign of the beta coefficient and was found significant for class 2, but was not significant for class 1. The negative elasticity value (-0.059), suggests that 1% increase in average flow, results in 5.9% reduction in the probability of a severe/fatal severity outcome. This means that high average traffic volumes result in less severe accidents, which was also found in Christoforou et al. (2010). Consequently, less congestion leads to more severe accidents and in that context, the presence of low number of heavy vehicles may worsen the situation, as more space for maneuvers is left. Moreover, considering that off-road collisions and collisions with fixed objects were found to result in more severe injuries, one might conclude that injury severities in Attica Tollway may not be a matter of interaction among road users.

The standard deviation of occupancy (Occ\_stdev\_30m\_up) was found to have mixed effects on severity, depending on the latent class, meaning that there is variation in the effect. The positive sign in the first latent class, shows that large variation of occupancy has higher probably of resulting in severe and fatal injuries of occupants. In general, traffic variations are linked to severe accidents (Yu and Abdel-Aty, 2014b), but in this is not supported for

latent class 2, where a negative sign of the beta coefficient is observed. Consequently, the final effect is based on the elasticity, where the positive elasticity value (-0.059), suggests that 1% increase in average flow, results in 5.9% decrease in the probability of a severe/fatal severity outcome.

## 5. Conclusions

The aim of this paper was to investigate road safety in motorways by utilizing high resolution traffic and weather data and other accident attributes. By applying finite mixture logit models, it was found that a number of traffic parameters such as truck proportion, average flow and standard deviation of occupancy, as well as other risk factors, such as accident type and engine size have a significant effect on the injury severity outcome of vehicle occupants. Moreover, this model accounted for the heterogeneity among two distinct groups of observations. For example, the impact of average flow and standard deviation of occupancy was not fixed across the two produced latent classes and diverse results were produced. However, the elasticity analysis revealed a negative effect of average flow and a positive effect of occupancy variation. On the other hand, high percentage of trucks consistently reduce severity levels of involved injured occupants. Collisions with fixed objects or run-off road collisions as well low engine size vehicles are associated with higher severity levels.

This paper contributes on the current knowledge, by having a specific consideration of real-time traffic and weather data when analyzing severity in motorways and also by applying finite mixture models. From a methodological point of view, the application of finite mixture models not only accounts for the unobserved heterogeneity but also proved capable of providing an understanding of the factors affecting occupant injury severity. More research is needed in order to incorporate also vulnerable road users such as motorcyclists, cyclists and pedestrians and further investigate the impact of real-time traffic and weather parameters.

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