Controlling Crest Vertical Curvature Rates Based on Variable Grade Stopping Sight Distance Calculation

Stergios Mavromatis, Nikiforos Stamatiadis, Basil Psarianos, and George Yannis

Stopping sight distance (SSD) is a key control element that directly affects the suggested values of crucial road design parameters. Although there is a significant difference in SSD values between upgrades and downgrades, many design policies ignore the grade effect during vehicle braking on variable grades. Such a case occurs during the determination of crest vertical curvature rates in which the relevant SSD values are extracted assuming leveled road geometry. This paper investigates a possible deficiency of such an approach with regard to cases in which the length of the vertical curve exceeds the control SSD values. SSD calculation on variable grades during the braking process was addressed through a recently developed process that related the point mass model and the laws of mechanics. For a wide range of design speed values, charts illustrating the required SSDs were drawn as a function of negative ending grade values related to the control crest vertical curve rates adopted by AASHTO. The process revealed numerous SSD shortage areas for which revised crest vertical curvature rates were provided to grant SSD adequacy throughout the vehicles' braking process. This paper also aimed to provide designers with ready-to-use vertical design tools associated with amended vertical curvature rates to AASHTO's road functional classification as a function of the crest vertical curve's exit grade value.

Sight distance is the length of roadway ahead that is visible to the driver (*1*). The minimum sight distance, known as stopping sight distance (SSD), is a highway geometric design element of fundamental importance. SSD must be provided at every point along the road surface, thus it affects critical road design parameters that directly impose economic considerations on both new road designs and road improvement projects (e.g., *1–4*).

AASHTO's *A Policy on Geometric Design of Highways and Streets* (commonly referred to as the Green Book) notes that for vertical curves, the grade effect is somewhat balanced and there is no need to adjust SSD because of grade (*1*). The Green Book also notes, however, that there is a significant difference in SSD values between upgrades and downgrades (*1*). Moreover, Green Book guidelines state that the minimum lengths of crest vertical curves, based on sight distance criteria, are generally satisfactory from the standpoint of safety, comfort, and appearance, which implies that the vertical curvature rate is adequately determined through the suggested maximum grade control values, at least with regard to normal design cases and eliminating areas such as decision areas (e.g., ramp exit gores).

The functional classification of the road controls the maximum grade values. As such, the objective of the paper is to investigate the sufficiency of the suggested AASHTO crest vertical curvature rates considering the grade control, since these rates are defined based on a level road surface.

S. Mavromatis, School of Civil Engineering and Surveying and Geoinformatics Engineering, Technological Educational Institute of Athens, 2 Agiou Spiridonos Street, Athens 12210, Greece. N. Stamatiadis, Department of Civil Engineering, University of Kentucky, 265 Raymond Building, Lexington, KY 40506-0281. B. Psarianos, School of Rural and Surveying Engineering, National Technical University of Athens, 9 Hiroon Polytechniou Street, Athens 15780, Greece. G. Yannis, School of Civil Engineering, National Technical University of Athens, 5 Hiroon Polytechniou Street, Athens 15773, Greece. Corresponding author: S. Mavromatis, stemavro@teiath.gr.

Transportation Research Record: Journal of the Transportation Research Board, No. 2521, Transportation Research Board, Washington, D.C., 2015, pp. 31–44.
DOI: 10.3141/2521-04

BACKGROUND

According to existing design policies, the SSD of a vehicle consists of two distance components: the distance traveled during a driver's perception–reaction time from the instant the brakes are applied and the distance traveled while braking until the vehicle stops (*1–4*). The SSD model adopted by the Green Book is represented in Equation 1:

$$\mathrm{SSD} = V_o t_{\mathrm{pr}} + \frac{V_o^2}{2g\left(\dfrac{a}{g} + s\right)} \tag{1}$$

where

V_o = vehicle initial speed (m/s),
t_{pr} = driver's perception–reaction time (s) [2.5 s (*1*)],
g = gravitational constant (m/s^2) [9.81 m/s^2 (32.2 ft/s^2)],
a = vehicle deceleration rate (m/s^2) [3.4 m/s^2 (11.2 ft/s^2) (*1*)], and
s = road grade (%/100) [(+) upgrades, (−) downgrades].

Current road design standards determine minimum lengths of crest vertical curves as well as the consequent rate of vertical curvature based on SSD provision (*1–4*). Equation 2 and Equation 3 illustrate the parameters utilized in determining the length of crest vertical curves, and the vertical curvature rate definition is shown in Equation 4:

$$L = \frac{(s_2 - s_1)\mathrm{SSD}^2}{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2} \qquad \mathrm{SSD} < L \tag{2}$$

$$L = 2\mathrm{SSD} - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{s_2 - s_1} \qquad \mathrm{SSD} > L \tag{3}$$

$$K = \frac{L}{s_2 - s_1} \tag{4}$$

where

K = vertical curvature rate (m),
L = length of vertical curve (m),
h_1 = driver eye height (m) [1.08 m (3.50 ft) (*1*)],
h_2 = object height (m) [0.60 m (2.00 ft) (*1*)], and
s_1, s_2 = grade values (%).

The values of K (m) derived for SSD < L also apply without significant difference for SSD > L.

Most of the current efforts to evaluate SSD adequacy are based on two-dimensional models. Moreover, such efforts present a fragmented approach (i.e., examination of single elements) in investigating the adequacy of SSD and may underestimate or overestimate the available sight distance and thus possibly lead to safety violations (*5*).

Use of the vertical profile is a common approach in determining the adequacy of SSD, and roadway geometry is typically evaluated to ensure proper SSD requirements. The approach, however, fails to examine the continuity of the vertical alignment, especially in crest curves and their exiting grades. This failure has been noted as a potential safety issue (*6, 7*). There has been very little, if any, work that examines the impact of exit grades on vertical curve length even though there is the potential for requiring different lengths for vertical crest curves when exit grades are considered.

Table 1 illustrates the adopted design control values (rounded values) in the Green Book for SSD and crest vertical curvature rates of various design speed values, based on the above equations (*1*). It should be noted here that these values are reflective of daylight conditions, since most recommendations from design policies are for daylight conditions unless otherwise developed (e.g., sag curves).

Equations 2 and 4 show that the delivered crest vertical curvature rate is not grade dependent. The grade effect is indirectly introduced from the SSD determination for which 0% is assumed according to AASHTO guidelines.

Other design guides, although they use the same equations, adopt control crest vertical curvature rates through various other considerations. In the German design guidelines for motorways, the SSD values used for the crest vertical curvature rate determination are for the most unfavorable (negative) grade values (*2*). The relevant Greek OMOE-X design guidelines, in cases of two-lane rural roads, introduce a safety margin of +10 km/h in the SSD calculation; for example, to determine the crest vertical curvature rate for a design speed of 60 km/h, the SSD value used refers to 70 km/h (*3*).

The possible deficiency of the current SSD determination approach as adopted in the Green Book will be examined further by introducing the grade effect during braking on crest vertical transitions.

TABLE 1 Design Control Values for SSD and Crest Vertical Curvature Rates

Metric			U.S. Customary		
V_{design} (km/h)	SSD (m)	K (m)	V_{design} (mph)	SSD (ft)	K (ft)
50	65	7	30	200	19
60	85	11	40	305	44
70	105	17	45	360	61
80	130	26	50	425	84
90	160	39	55	495	114
100	185	52	60	570	151
110	220	74	70	730	247
120	250	95	75	820	312
130	285	124	80	910	384

BRAKING CALCULATION ON VARIABLE GRADES

The current road design practices, through Equation 1, sufficiently address the grade effect during the SSD procedure. However, the braking distance calculation for crest or sag curves that have variable grades is based on rather balanced assumptions. In most cases, the algebraic mean grade value of the tangents preceding and succeeding the vertical curve is adopted. This concept, however, fails to deliver the actual braking distances for all cases in which the braking distance is less than the vertical curve length.

The work presented here on the evaluation of the effect of the variable grade during the braking process is based on a practice recently developed by the authors (*7*). In this practice, simple considerations based on the laws of mechanics (Equations 5 and 6) were applied, assuming time steps of 0.01 s, to determine both the instantaneous vehicle speed and pure braking distance (SSD minus distance traveled during driver's perception–reaction time).

$$V_{i+1} = V_i - g\left(\frac{a}{g} + s\right)t \tag{5}$$

$$\mathrm{BD}_i = V_i t - \frac{1}{2}g\left(\frac{a}{g} + s\right)t^2 \tag{6}$$

where

V_i = vehicle speed at specific station i (m/s),
V_{i+1} = vehicle speed reduced by the deceleration rate t = 0.01 s (m/s),
t = time fragment (0.01 s),
s = road grade in i position (%/100) [(+) upgrades, (−) downgrades], and
BD_i = pure braking distance (m).

By applying Equations 5 and 6 subsequently there is a sequence value $i = k - 1$ in which V_k becomes equal to zero. The corresponding value of $\sum \mathrm{BD}_{k-1}$ represents the vehicle's total pure braking distance for the initial value of vehicle speed. The variable grade SSD is produced by adding the final pure braking distance to the distance traveled during the driver's perception–reaction time (first component of Equation 1) as follows:

$$\mathrm{SSD} = V_o t_{pr} + \sum \mathrm{BD}_{k-1} \tag{7}$$

where $\sum \mathrm{BD}_{k-1}$ is total vehicle pure braking distance for the initial value of vehicle speed (m).

The proposed approach, although similar to Equation 1, is more appropriate since the actual grade variation is considered.

CREST VERTICAL CURVATURE RATE ADEQUACY INVESTIGATION

The Green Book's potential inadequacy on the suggested vertical curvature rates must be addressed in the negative grade area since, on one hand, downgrades increase the SSD for a vehicle and, on the other, the current vertical curvature rate definition is extracted assuming flat vertical geometry.

In the following paradigm, an investigation of the crest vertical curvature rate that sufficiency assumes 70 km/h (45 mph) design speed is carried out by defining the actual SSD values along two specified positions. Figure 1 illustrates the length of the consequent vertical curve adopted by the Green Book [K = 17 m (61 ft)] for which the approach and exit grade values are set to +10% and −10%, respectively. Two cases of vehicle braking are shown as follows:

Case 1. The braking procedure begins at the starting point of the vertical curve (s = 10%) where SSD = 96.3 m (316 ft) and

Case 2. The braking procedure begins at the midpoint of the vertical curve (s = 0%) where SSD = 113.0 m (370 ft).

In both cases the SSD definition was based on the calculation procedure described previously. As expected, the SSD value in Case 2 is greater than not only the relevant SSD value in Case 1, but the SSD design control of 105 m (360 ft) for the 70 km/h (45 mph) design speed (in Table 1) as well. This situation indicates that there are areas along the vertical curve t in which the braking procedure requires greater SSD values and thus an increase of the crest vertical curvature rate in these cases seems indispensable.

Figure 2*a* illustrates certain SSD values extracted for a vehicle braking under 70 km/h (design speed) and the corresponding K value (K = 17 m). The relevant values drawn for the equivalent speed (approximately) in U.S. customary units are shown in Figure 2*b*. The SSD values of Figure 2 were drawn arranging the total braking procedure to be performed entirely inside the crest vertical curve. The horizontal axes in Figure 2 show the grade value within the crest vertical curve where the vehicle is supposed to stop.

The data show that as the downgrade becomes steeper, the required SSDs increase accordingly. In cases where the required SSDs exceed the relevant values suggested in the Green Book, the suggested K values are introduced in the secondary vertical axes. For example, in Figure 2*b* when an ending grade of a crest vertical curve is set to −8% and the length of the variable grade area is above 400 ft, the minimum crest vertical curvature rate value must be approximately K = 75 ft, as opposed to the currently suggested K = 61 ft, to grant SSD adequacy.

To illustrate the above procedure as more integrated, Figures 3–10 show the suggested crest vertical curvature rates based on the vertical curve's ending grade value for design speed values of 50 km/h, 60 km/h, 80 km/h, 90 km/h, 100 km/h, 110 km/h, 120 km/h, and 130 km/h (paired with the equivalent U.S. customary units, respectively). These figures assess the braking effect on steep (mostly) variable downgrades and thus deliver ready-to-use crest vertical curvature rate values for designers.

At first glance of Figures 2–10, one would expect a similar variation between the values outlined in Case 1 and Case 2. However, even for exactly equivalent speed values, random rounding concepts result in slight variation on the delivered SSD and K values, respectively.

The ending grades used in Figures 2–10 were drawn based on the grade control criteria according to AASHTO's road functional classification. For example, in Figure 3, the selection of grade values up to 14% for 50 km/h (80 mph) design speed refers to mountainous local rural roads, as identified in the Green Book.

Figures 2–10 show that there is a horizontal variation between the intersection point (red line intersection) of the SSDs and the intersection point (blue line intersection) of the relevant K values. Theoretically, intersection points should be at the same ending grade value, since both refer to the same value. This, however, is not the case: the crest vertical curvature rates calculation formula based on

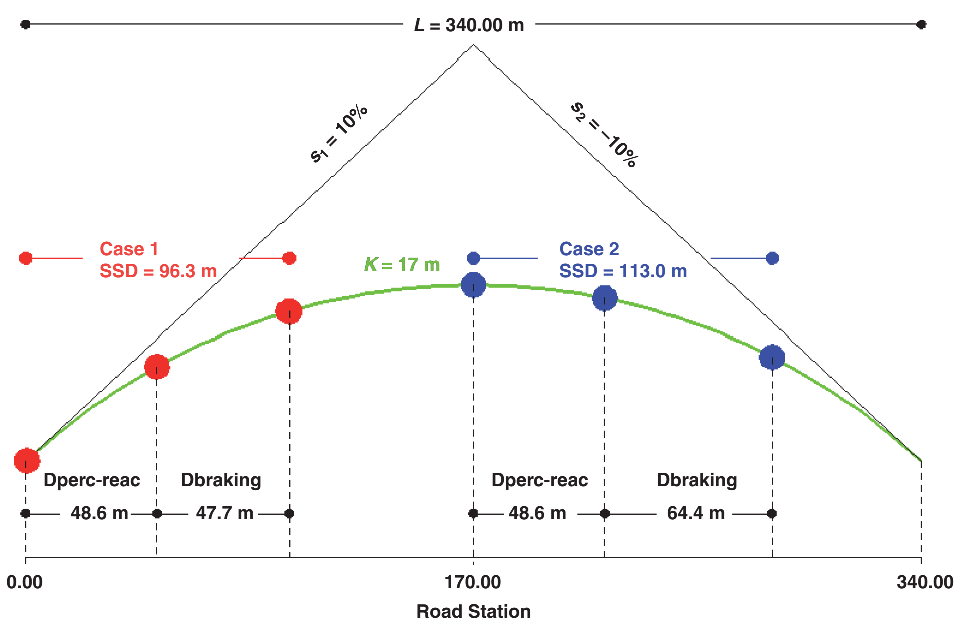


FIGURE 1 Discriminating cases of SSD variation on AASHTO-recommended crest vertical curvature rate V_{design} = 70 km/h (45 mph), K = 17 m (61 ft) (Dperc–reac = driver perception–reaction distance; Dbraking = driver braking distance).

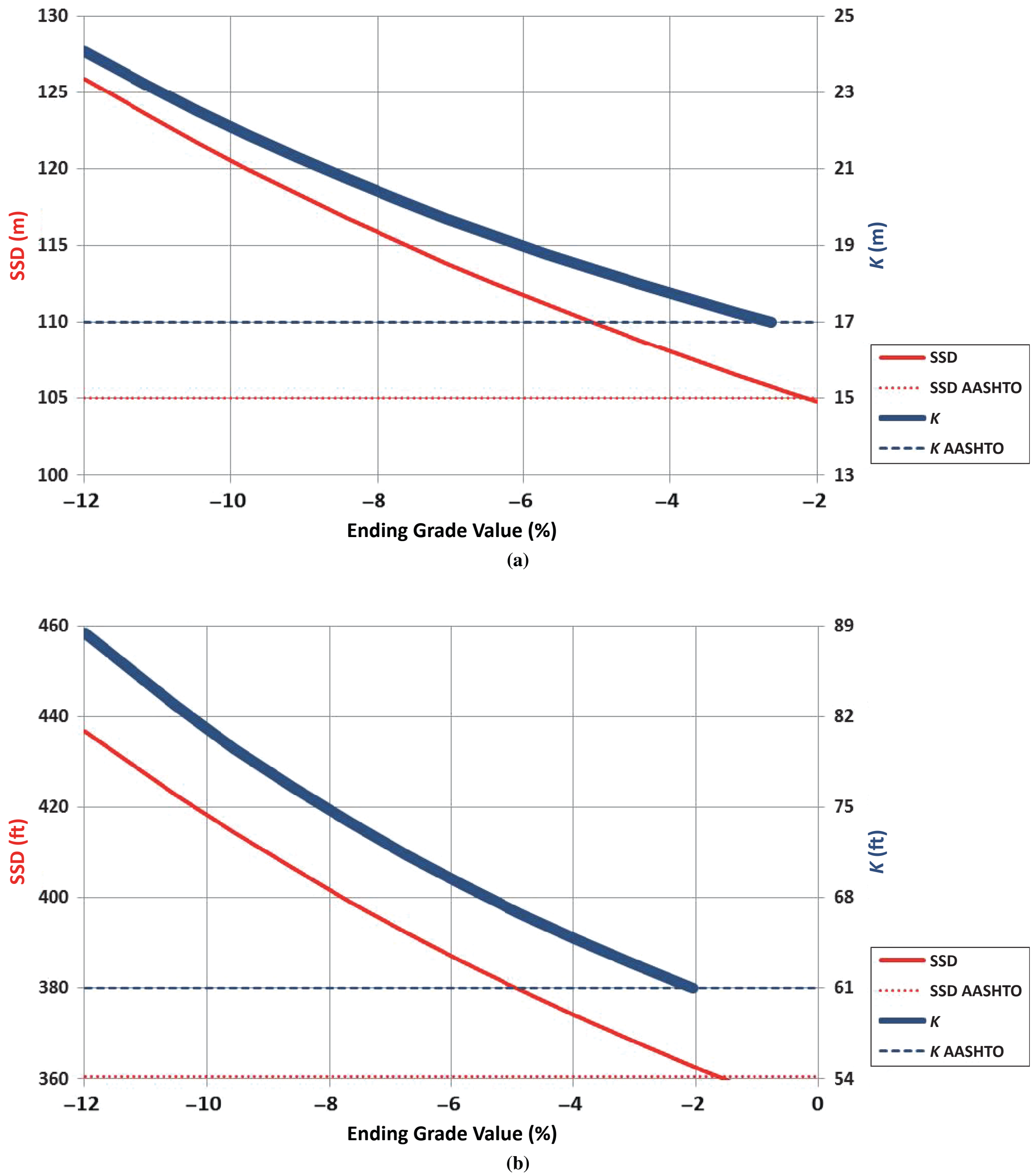
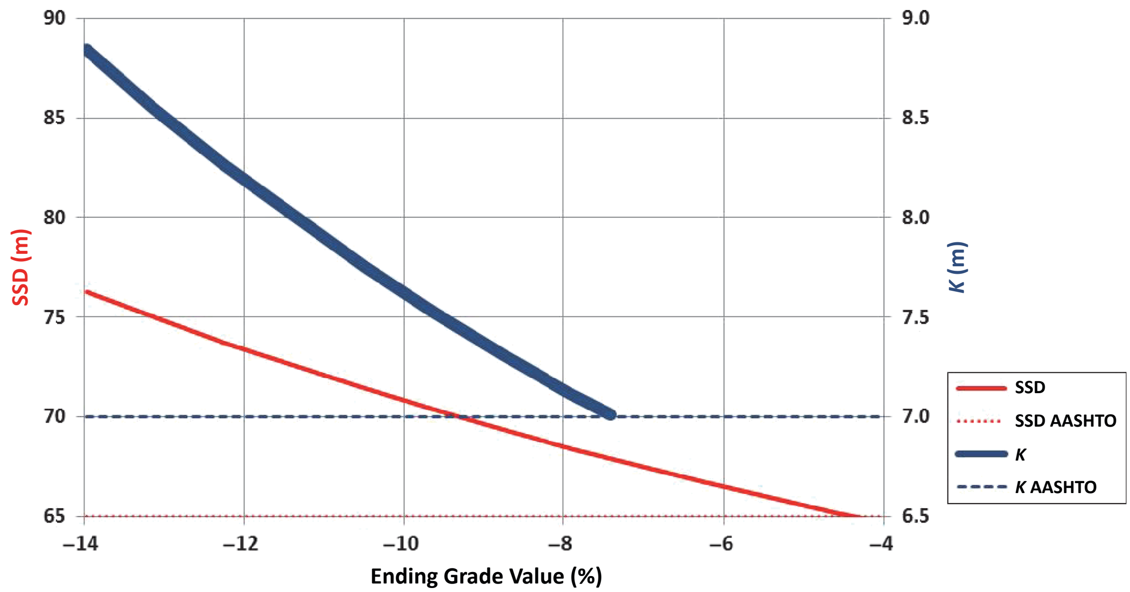
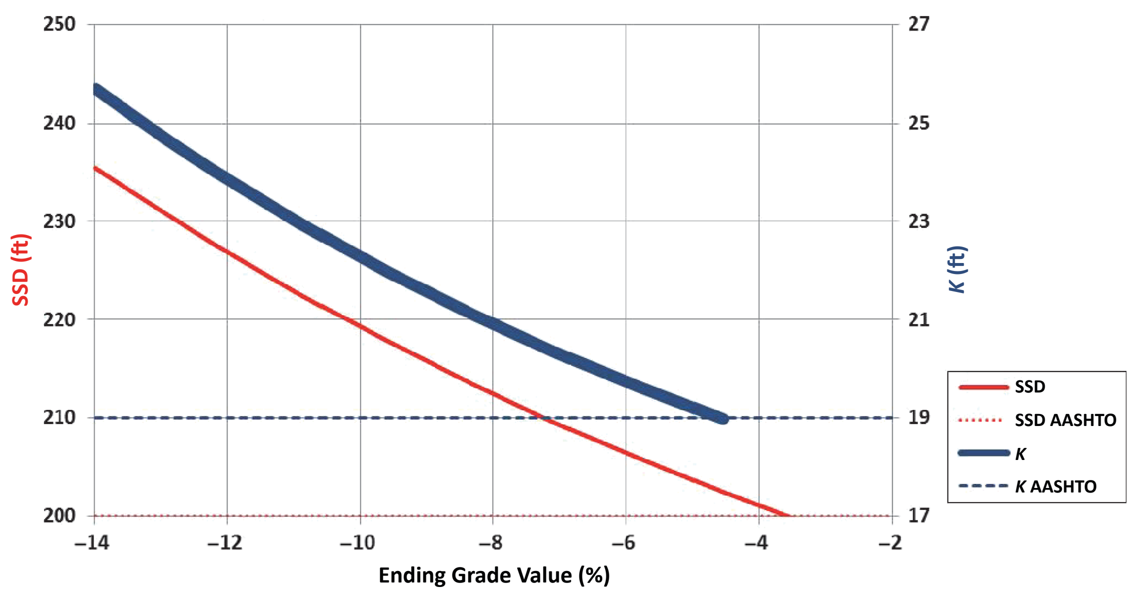


FIGURE 2 Suggested crest vertical curvature rate values based on SSD adequacy: (*a*) V_{design} = 70 km/h and (*b*) V_{design} = 45 mph.



(a)



(b)

FIGURE 3 Suggested crest vertical curvature rate values based on SSD adequacy: (*a*) V_{design} = 50 km/h and (*b*) V_{design} = 30 mph.

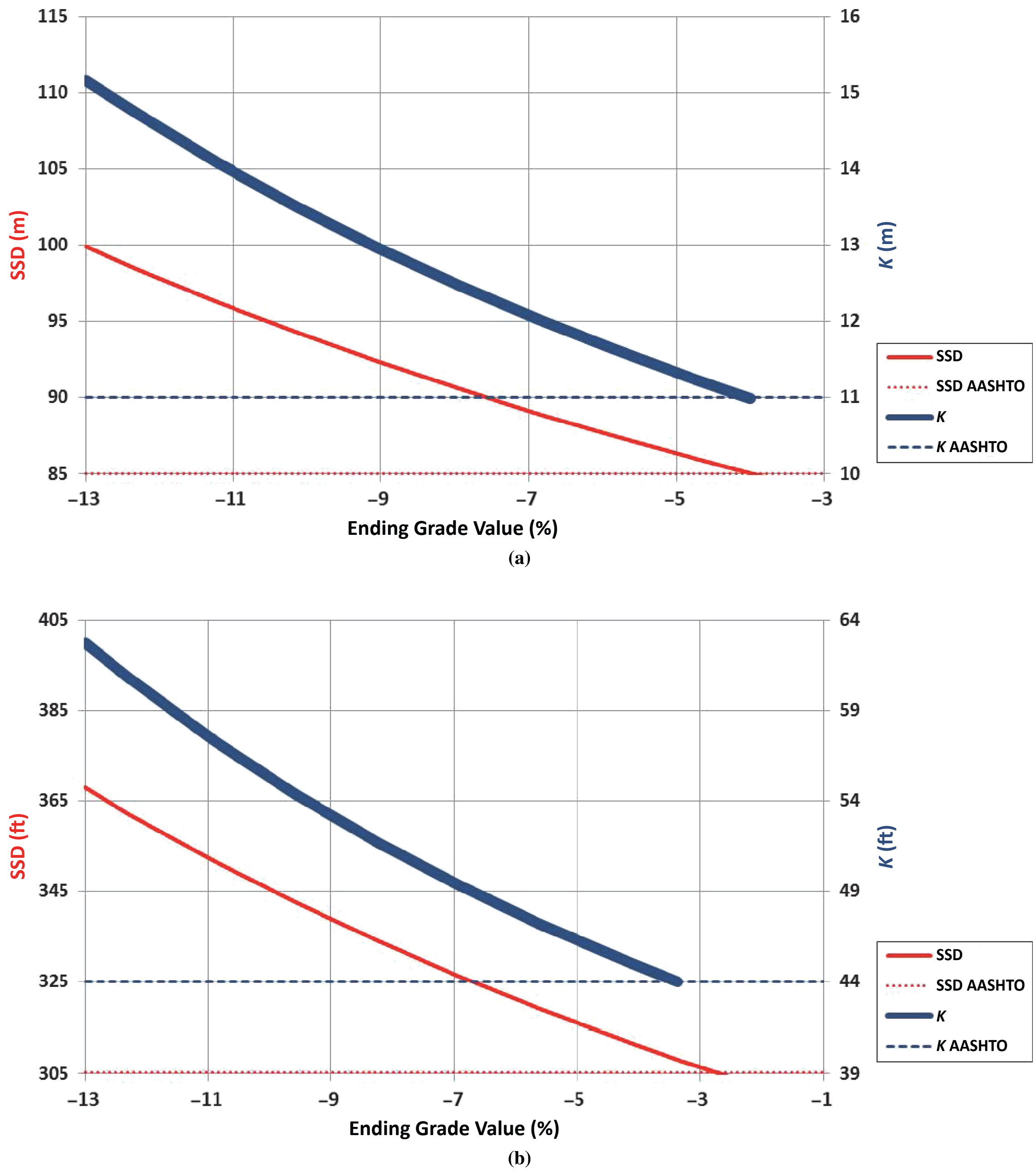


FIGURE 4 Suggested crest vertical curvature rate values based on SSD adequacy: (*a*) V_{design} = 60 km/h and (*b*) V_{design} = 40 mph.

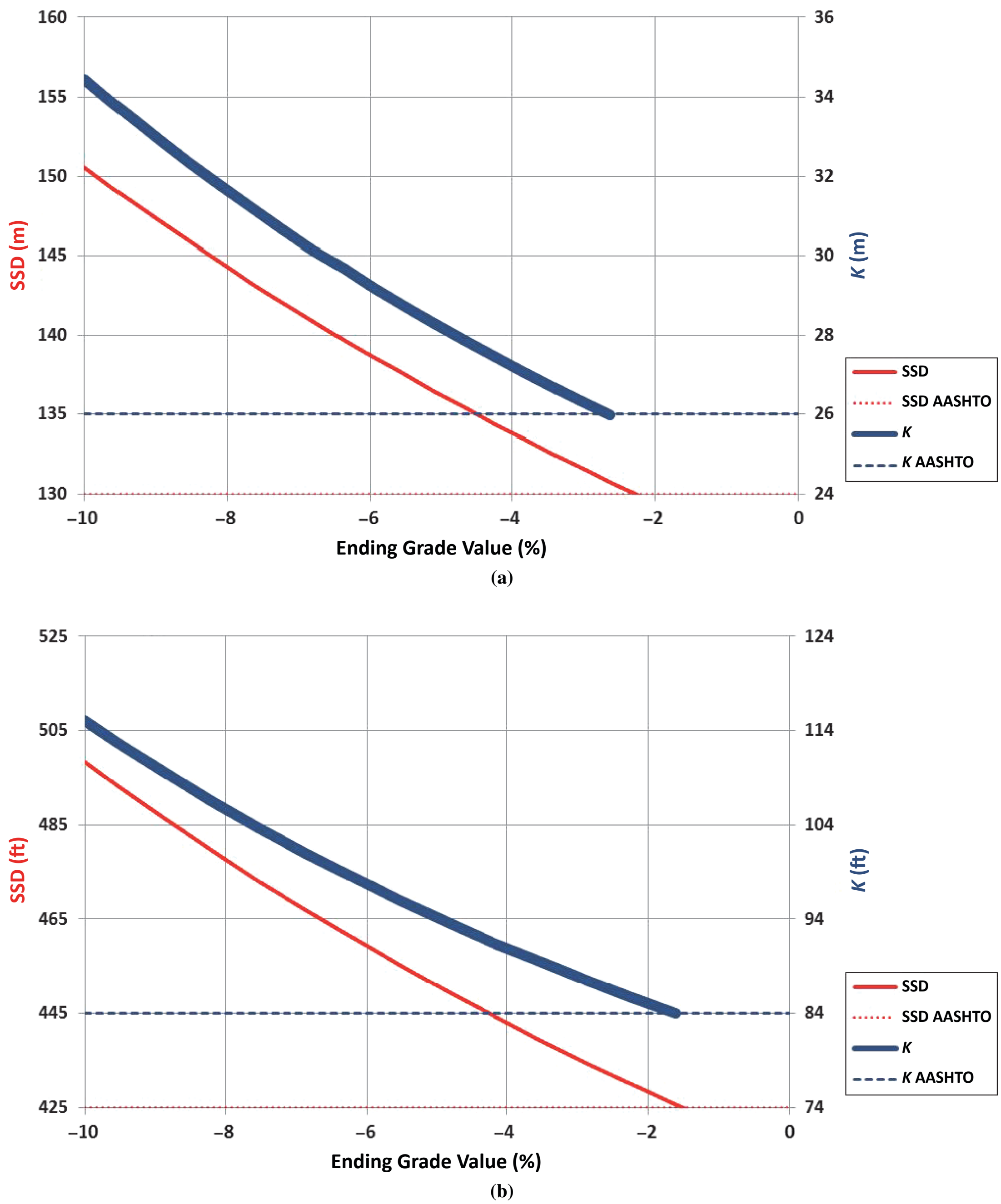


FIGURE 5 Suggested crest vertical curvature rate values based on SSD adequacy: (*a*) V_{design} = 80 km/h and (*b*) V_{design} = 50 mph.

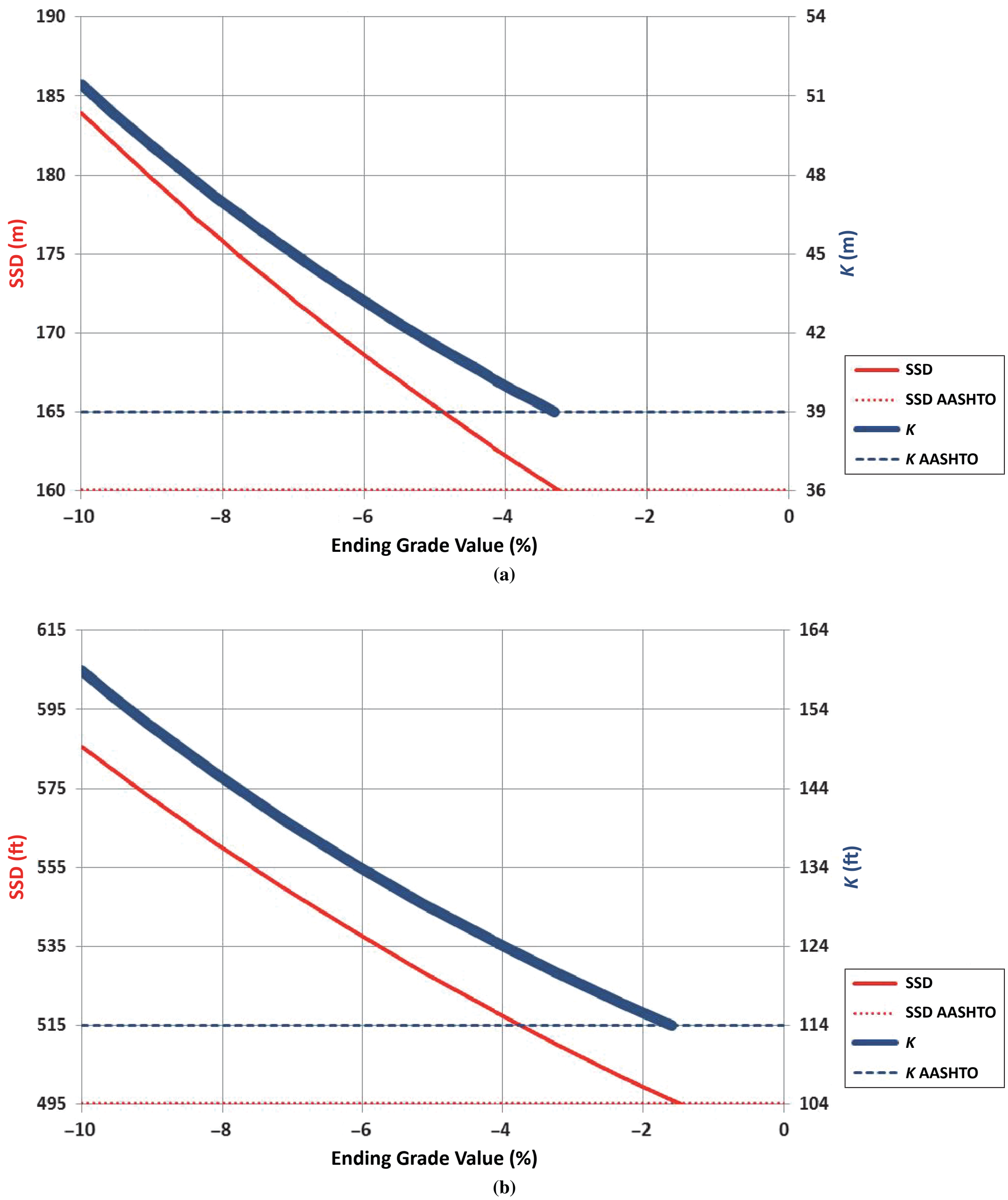


FIGURE 6 Suggested crest vertical curvature rate values based on SSD adequacy: (*a*) V_{design} = 90 km/h and (*b*) V_{design} = 55 mph.

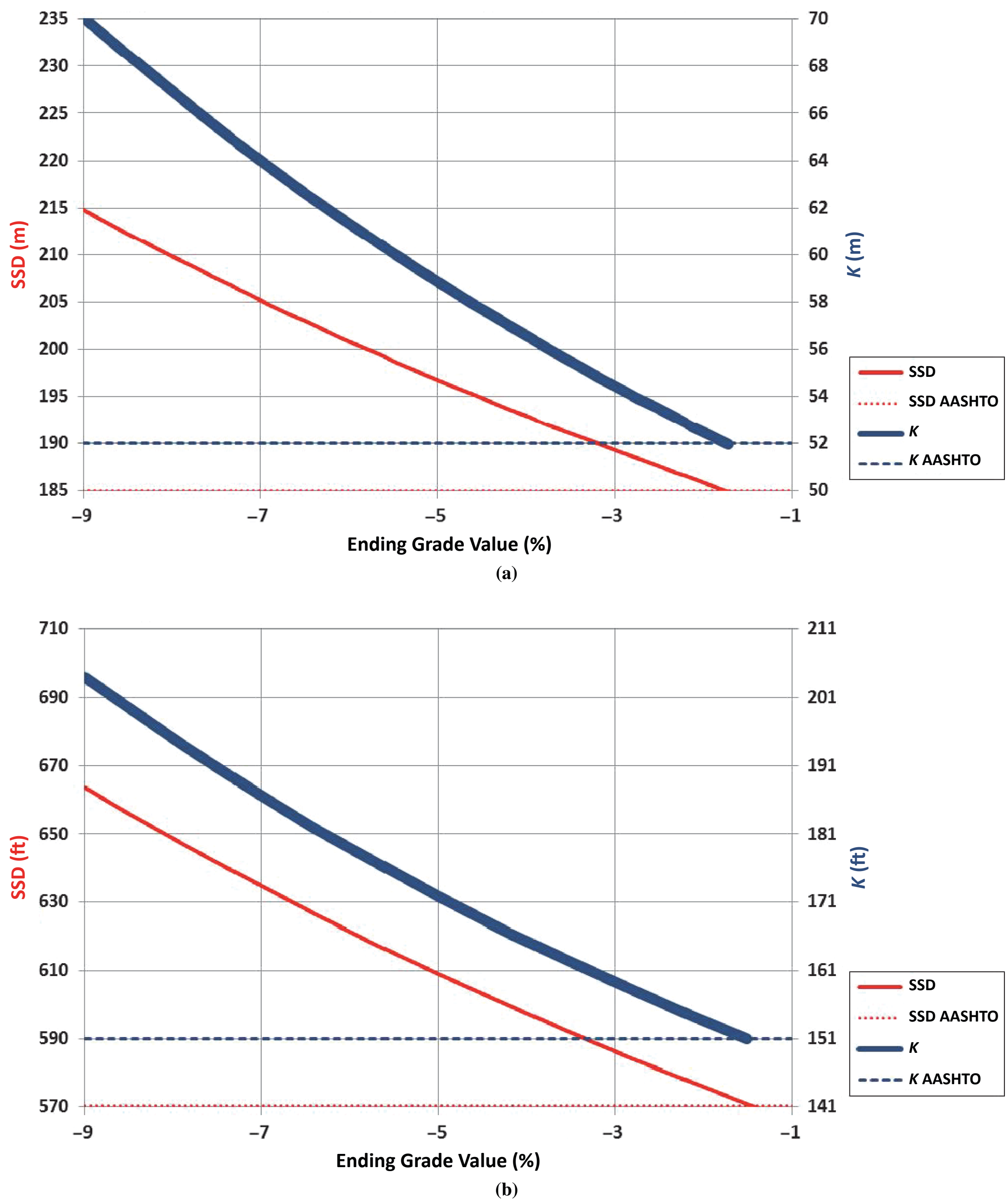
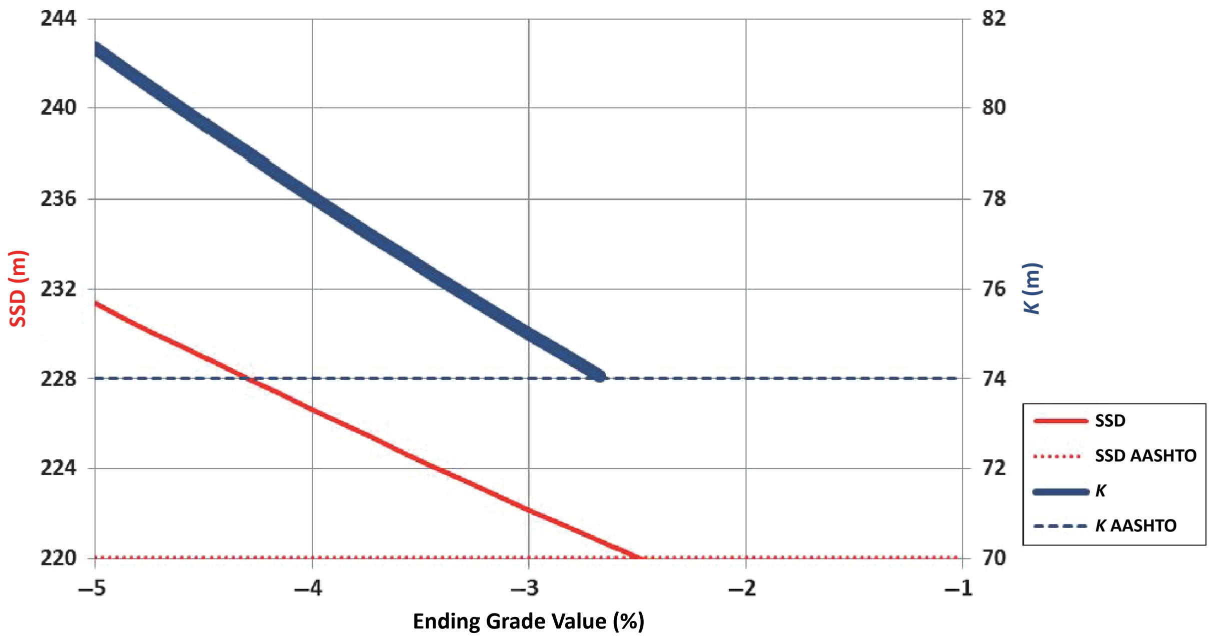
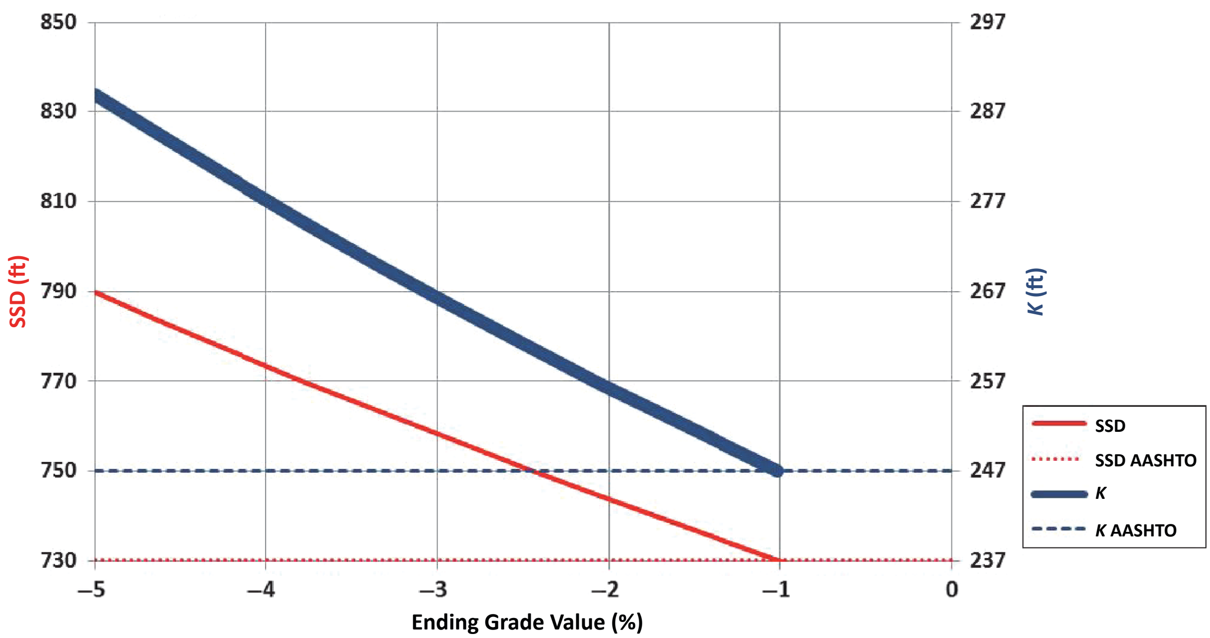


FIGURE 7 Suggested crest vertical curvature rate values based on SSD adequacy: (*a*) V_{design} = 100 km/h and (*b*) V_{design} = 60 mph.

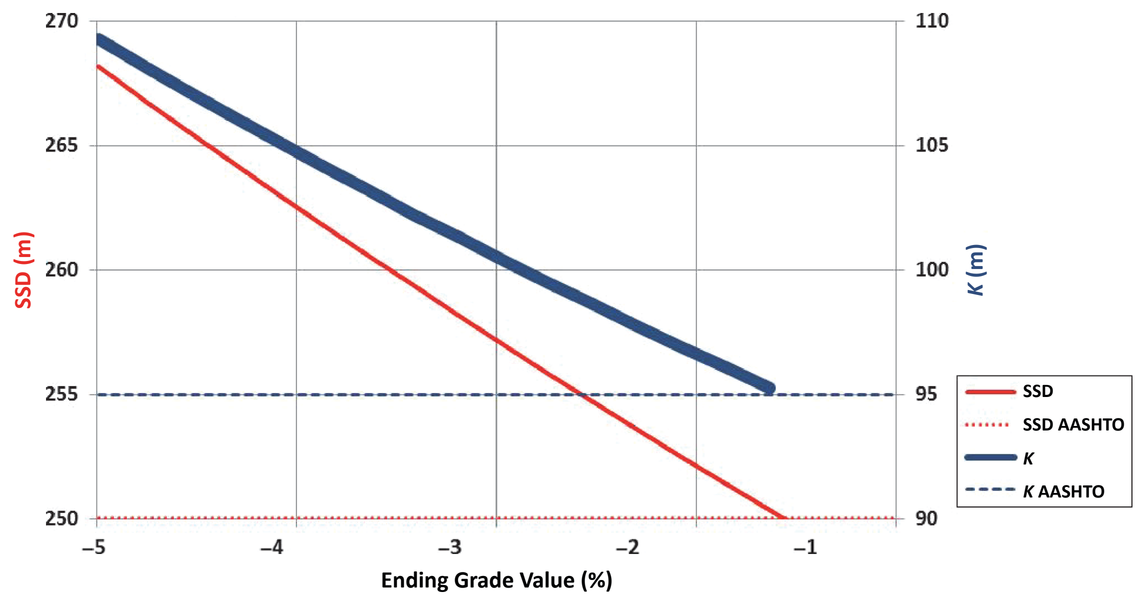


(a)

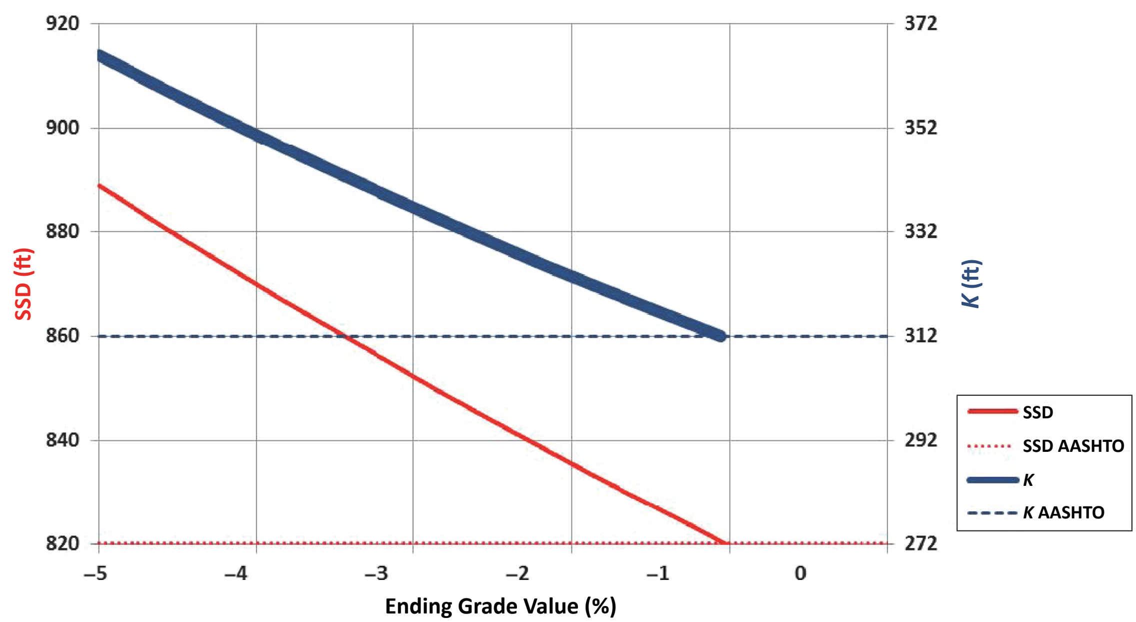


(b)

FIGURE 8 Suggested crest vertical curvature rate values based on SSD adequacy: (*a*) V_{design} = 110 km/h and (*b*) V_{design} = 70 mph.



(a)



(b)

FIGURE 9 Suggested crest vertical curvature rate values based on SSD adequacy: (*a*) V_{design} = 120 km/h and (*b*) V_{design} = 75 mph.

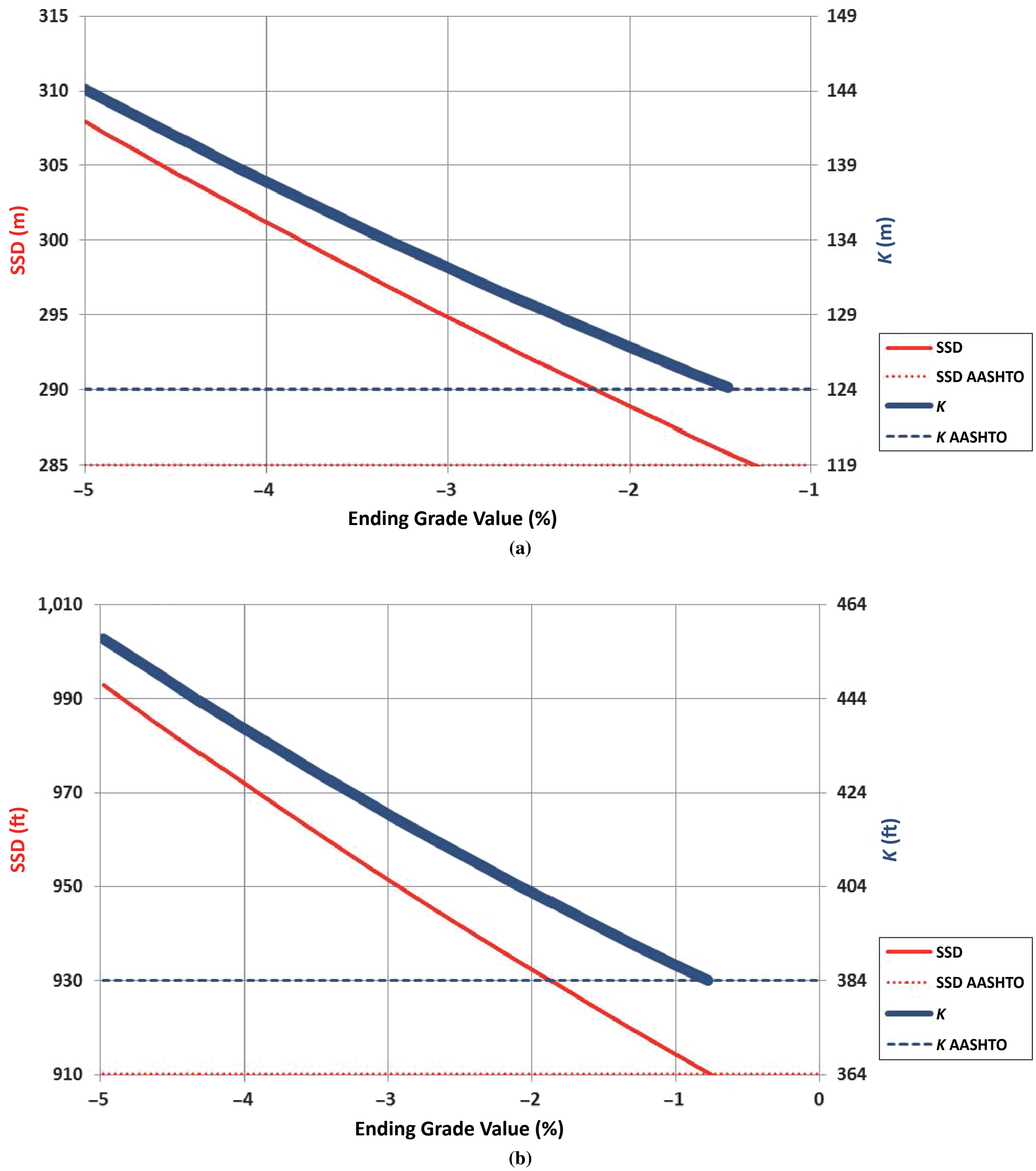


FIGURE 10 Suggested crest vertical curvature rate values based on SSD adequacy: (*a*) V_{design} = 130 km/h and (*b*) V_{design} = 80 mph.

Equations 2 and 4 deliver values that are rounded for design. As an example, for the following two extreme cases:

- V_{design} = 50 km/h. The K value is calculated 6.4 m but introduced as 7.0 m in the Green Book (*1*).
- V_{design} = 90 km/h. The K value is calculated 38.9 m but introduced as 39.0 m in the Green Book (*1*).

In the first case, the adopted K value for design is increased by 9.4%, whereas in the second, the value is increased only by 0.3%. This explains the horizontal variation between the two intersection points.

CORRELATION BETWEEN SUGGESTED CREST VERTICAL CURVATURE RATE VALUES AND ROAD FUNCTIONAL CLASSIFICATION

The above analysis reveals an SSD inadequacy for the downgrade area of steep grades when control crest vertical curvature rates based on AASHTO design guidelines are used; however, as each road class is associated with a particular grade range, its impact on the above process is ignored.

Table 2 shows the correlation between AASHTO's road functional classification and the amended crest vertical curvature rates calculated here. (It should be noted that the table uses only a single design speed of 80 km/h (50 mph) for all possible functional classification categories.) Table 2 shows ready-to-use values of crest vertical curvature rates based on the desired exiting grade value of the design. The illustrated crest curvature rates refer to negative exit grade values. The values are based on the plots shown in Figure 5 and can be applied in cases where the length of the vertical curve exceeds the required SSD values as shown. The gaps in the suggested crest vertical curvature rates indicate that the maximum grade value of the certain road class is lower.

CONCLUSIONS

This paper investigates the consequences of the Green Book guidelines that adopt minimum crest vertical curvature rates based on leveled grade values.

TABLE 2 Suggested Crest Vertical Curvature Rate Values Based on Roadway's Functional Classification for V_{design} = 80 km/h (V_{design} = 50 mph)

		Exit Grade Value							
Type of Terrain	AASHTO	–3%	–4%	–5%	–6%	–7%	–8%	–9%	–10%
Local rural									
Level		—	—	—	—	—	—	—	—
Rolling (m)	26	27	28	29	30	31	32	33	35
		(*L* > 132)	(*L* > 134)	(*L* > 136)	(*L* > 139)	(*L* > 142)	(*L* > 144)	(*L* > 147)	(*L* > 151)
Mountainous	84	88	91	95	98	102	106	111	115
(ft)		(*L* > 436)	(*L* > 443)	(*L* > 451)	(*L* > 459)	(*L* > 468)	(*L* > 478)	(*L* > 488)	(*L* > 498)
Rural collectors									
Level		—	—	—	—	—	—	—	—
Rolling (m)	26	27	28	29	30	31	32	33	—
		(*L* > 132)	(*L* > 134)	(*L* > 136)	(*L* > 139)	(*L* > 142)	(*L* > 144)	(*L* > 147)	—
Mountainous	84	88	91	95	98	102	106	111	—
(ft)		(*L* > 436)	(*L* > 443)	(*L* > 451)	(*L* > 459)	(*L* > 468)	(*L* > 478)	(*L* > 488)	—
Rural arterials									
Level	—	—	—	—	—	—	—	—	—
Rolling (m)	26	27	28	29	30	31	—	—	—
		(*L* > 132)	(*L* > 134)	(*L* > 136)	(*L* > 139)	(*L* > 142)	—	—	—
Mountainous	84	88	91	95	98	102	—	—	—
(ft)		(*L* > 436)	(*L* > 443)	(*L* > 451)	(*L* > 459)	(*L* > 468)	—	—	—
Urban collectors									
Level		—	—	—	—	—	—	—	—
Rolling (m)	26	27	28	29	30	31	32	33	35
		(*L* > 132)	(*L* > 134)	(*L* > 136)	(*L* > 139)	(*L* > 142)	(*L* > 144)	(*L* > 147)	(*L* > 151)
Mountainous	84	88	91	95	98	102	106	111	115
(ft)		(*L* > 436)	(*L* > 443)	(*L* > 451)	(*L* > 459)	(*L* > 468)	(*L* > 478)	(*L* > 488)	(*L* > 498)
Urban arterials									
Level		—	—	—	—	—	—	—	—
Rolling (m)	26	27	28	29	30	31	32	33	—
		(*L* > 132)	(*L* > 134)	(*L* > 136)	(*L* > 139)	(*L* > 142)	(*L* > 144)	(*L* > 147)	—
Mountainous	84	88	91	95	98	102	106	111	—
(ft)		(*L* > 436)	(*L* > 443)	(*L* > 451)	(*L* > 459)	(*L* > 468)	(*L* > 478)	(*L* > 488)	—
Freeways									
Level		—	—	—	—	—	—	—	—
Rolling (m)	2	27	28	29	30	—	—	—	—
		(*L* > 132)	(*L* > 134)	(*L* > 136)	(*L* > 139)	—	—	—	—
Mountainous	84	88	91	95	98	—	—	—	—
(ft)		(*L* > 436)	(*L* > 443)	(*L* > 451)	(*L* > 459)	—	—	—	—

NOTE: *L* refers to length of vertical curve; values in both metric (m) and U.S. customary units (ft). — indicates that maximum grade value of certain road class is lower.

The SSD calculation on variable grades during the braking process was initially addressed through an earlier approach based on the point mass model and the laws of mechanics. This process resulted in determining that the negative grade area of crest vertical curves, as expected, increases the calculated SSD values.

Subsequently, for a wide range of design speed values, charts of the required SSDs were drawn as a function of various ending grade values, based on control crest vertical curve rates, as adopted by AASHTO. Since various SSD shortage areas appeared in the charts, amended crest vertical curvature rates were provided to grant SSD adequacy throughout the braking process. The resulting crest vertical rates apply for cases in which the length of the vertical curve exceeds the calculated SSD values.

Immediate implementation of this approach would provide designers with ready-to-use revised crest vertical curvature rates, based on the desired exiting grade value of the design that is in accordance with AASHTO's roadway functional classification. However, further analysis is required to determine the potential effects of combined horizontal–vertical alignment requirements, and this may necessitate a revision of the proposed approach and ready-to-use-tool.

Moreover, additional qualitative research seems necessary to evaluate parameters of SSD (braking on curves, braking within an antilock braking system, friction coefficient, etc.) and to reflect current vehicle dynamics trends and thus simulate the braking procedure more realistically. The human factor should also be considered as it might impose additional restrictions and consequently influence the braking process to an extent beyond the perception–reaction procedure and friction reserve that are already part of the braking process.

Finally, it is also necessary to underline that the parameters used in this paper (e.g., speed values, perception–reaction time) apply to daylight driving conditions. For nighttime driving conditions, vehicle speed values are 6–15 km/h less and the geometry of the road view changes (*8*).

REFERENCES

1. *A Policy on Geometric Design of Highways and Streets,* 6th ed. AASHTO, Washington, D.C., 2011.
2. *Guidelines for the Design of Freeways* (in German). Research Society for Roads and Transportation Association (FGSV), Cologne, Germany, 2008.
3. *Guidelines for the Design of Road Projects, Part 3, Alignment (OMOE-X).* Ministry of Environment, Regional Planning, and Public Works, Athens, Greece, 2001.
4. *Instrucción de Carreteras, Norma 3.1-IC Trazado.* Ministry of Public Works and Transport, Madrid, Spain, 2000.
5. Hassan, Y., S. M. Easa, and A. O. Abd El Halim. Design Considerations for Combined Highway Alignments. *Journal of Transportation Engineering,* Vol. 123, No. 1, 1997, pp. 60–68.
6. Moreno Chou, A., V. Perez, A. Garcia, and M. Rojas. Optimal 3-D Coordination to Maximize the Available Stopping Sight Distance in Two-Lane Roads. Presented at 4th International Symposium on Highway Geometric Design, Valencia, Spain, 2010.
7. Mavromatis S., S. Palaskas, and B. Psarianos. Continuous Three-Dimensional Stopping Sight Distance Control on Crest Vertical Curves. *Advances in Transportation Studies,* Special Issue 1, 2012, pp. 17–30.
8. Malakatas, K. *Operating Speed Predicting Model on Two-Lane Rural Roads During Night-Time.* MS thesis. National Technical University of Athens, Greece, 2012.

The Standing Committee on Geometric Design peer-reviewed this paper.