ATS: controlling sag vertical curvature rates based on variable grade stopping sight distance calculation

S. Mavromatis[1] N. Stamatiadis[2]
B. Psarianos[3] G. Yannis[4]

[1]School of Civil Engineering and Surveying & Geoinformatics Engineering
Technological Educational Institute of Athens
2 Agiou Spiridonos Str., 12210 Athens, Greece
email: stemavro@teiath.gr

[2]College of Engineering
University of Kentucky
265 Raymond Bldg.
Lexington, KY 40506-0281
e-mail: nick.stamatiadis@uky.edu

[3]School of Rural & Surveying Engineering
[4]School of Rural & Surveying Engineering
National Technical University of Athens
9 Hiroon Polytechniou Str., 15780 Athens, Greece
email: psari@survey.ntua.gr, geyannis@central.ntua.gr

Abstract

Stopping Sight Distance (SSD) is a key control element which directly affects the suggested values of crucial road design parameters. Although there is a significant difference in SSD values between upgrades and downgrades, many Design Policies ignore the grade effect during vehicle braking on variable grades. Such a case is experienced during the determination of sag vertical curvature rates where the relevant SSD values are extracted assuming leveled road geometry. The paper investigates possible deficiency of this approach, regarding cases where the length of the vertical curve exceeds the control SSD values.

The authors addressed the SSD calculation on variable grades during the braking process through a recently developed process that relates the point mass model and the laws of mechanics.

For a wide range of design speed values, charts illustrating the required SSDs were drawn as a function of negative entering grade values related to control sag vertical curve rates, as adopted by AASHTO. The process revealed numerous SSD shortage areas, where the authors provided revised sag vertical curvature rates, in order to grant SSD adequacy throughout the vehicles' breaking process.

Furthermore, the authors aiming to provide the designers with ready-to-use vertical design tool associated the amended vertical curvature rates to AASHTO's road functional classification, as a function of the sag vertical curve's entering grade value.

Keywords – stopping sight distance, sag vertical curvature rate, entering grade value

1. Introduction and problem statement

Sight distance is the length of roadway ahead that is visible to the driver [1]. The minimum sight distance known as Stopping Sight Distance (SSD), is a highway geometric design element of fundamental importance. SSD must be provided at every point along the road surface, thus affecting critical road design parameters which directly impose economic considerations on both new road designs as well as road improvement projects [e.g. 1, 2, 7, 8].

The American Association of State Transportation Officials (AASHTO) design guide titled "A Policy on Geometric Design of Highways and Streets" (commonly referred to as the Green Book) notes that, regarding sag vertical curves, although at least four different criteria for establishing lengths of sag vertical curves are recognised, namely; headlight sight distance, passenger comfort, drainage control and general appearance, the first criterion (headlight sight distance) seems to prevail. Moreover, the utilisation of headlight sight distance to establish design vales for a range of lengths of sag vertical curves is recommended, *as the most logical criterion for general use* [1].

In current practice, control lengths of vertical curves for sag conditions are based on SSD values. However, as noted in the Green Book [1], these values assume level road surface although significant differences in SSD values between upgrades and downgrades are also reported. Therefore, based on a recently developed process by the authors for crest vertical curves [6], the objective of the paper is to investigate the sufficiency of the suggested sag vertical curvature rates by AASHTO from the grade control point of view.

2. Background

According to existing design policies [e.g. 1, 2, 7, 8], the SSD of a vehicle consists of two distance components: the distance travelled during driver's perception – reaction time to the instant the brakes are applied and the distance while braking to stop the vehicle. For example, the SSD model adopted by the Green Book is represented by Equation (1).

$$SSD = V_o t_{pr} + \frac{V_o^2}{2g(\frac{a}{g} + s)} \qquad (1)$$

where :
V_o (m/sec) : vehicle initial speed
t_{pr} (sec) : driver's perception – reaction time [2.5sec; AASHTO, 2011]
g (m/sec^2) : gravitational constant [9.81m/sec^2 (32.2ft/sec^2)]
a (m/sec^2) : vehicle deceleration rate [3.4m/sec^2 (11.2ft/sec^2); AASHTO, 2011]
s (%/100) : road grade [(+) upgrades, (-) downgrades]

Current road design standards [e.g. 1, 2, 7, 8] determine minimum lengths of sag vertical curves as well as the consequent rate of vertical curvature based on SSD provision, where night driving seems to be the critical situation. Based once again on the Green Book [1]: … *for drivers to see the roadway ahead, a sag vertical curve should be long enough that the light beam distance is approximately the same as the SSD,* thus implying that the vehicle's light beam sight distance is the basis for determining sag vertical curves.

The vehicle's light beam is dependent on the position of the headlights and the direction of the light beam, or more specifically on the headlight height and the upward divergence of the light beam from the longitudinal axis of the vehicle.

The parameters utilized in determining the length of sag vertical curves (L) are illustrated in Equation (2) and Equation (3), where the vertical curvature rate definition (K) is shown through Equation (4). Similar to the case of crest vertical curves, the values of K derived for SSD<L, apply without significant difference also for the case SSD>L.

Accordingly, in the equations below and for various design speeds it is suggested to use the values determined for SSDs, which however refer to level grade.

$$L = \frac{(s_2 - s_1)SSD^2}{200[h + SSD\tan(\varepsilon)]} \qquad SSD<L \qquad (2)$$

$$L = 2SSD - \frac{200[h + SSD\tan(\varepsilon)]}{s_2 - s_1} \qquad SSD>L \qquad (3)$$

$$K = \frac{L}{s_2 - s_1} \qquad (4)$$

where:

K : vertical curvature rate (m)

L : length of sag vertical curve (m)

SSD : stopping sight distance (m)

h : headlight height (m) [0.60m (2.00ft); AASHTO 2011]

ε : upward divergence of the light beam from the longitudinal axis of the vehicle ($\varepsilon=1^o$)

s_1, s_2 : grade values (%)

Most of the current efforts to evaluate SSD adequacy are based on 2-Dimensional models. Moreover, such efforts present a fragmented approach (e.g. examination of single elements) in investigating the adequacy of SSD and may underestimate or overestimate the available sight distance and thus possibly lead to safety violations [3].

The use of the vertical profile is a common approach in determining adequacy of SSD and typically roadway geometry is evaluated to ensure proper SSD requirements. This approach however fails to examine the continuity of the vertical alignment especially in sag curves and their entering grades. This issue, at least regarding crest vertical curves, was noted in the past as a potential safety issue [5, 9].

There has been little work on this topic even though there is the potential for requiring different lengths for crest and sag vertical curves when exit and entrance downgrades respectively are considered. For example, according to previous research, a mathematical integration procedure was developed [10], further improved [11] and generalised for both crest and sag vertical curves [12], based on the average grade over the braking distance.

More specifically, the latter research [12] delivers minimum lengths of crest and sag vertical curves for selected approach and exit grade values. However, based on AASHTO 2011 design guidelines… *"it is convenient to express the design control in terms of the K rate for all values of approach and exit grades"*.

Therefore the present paper, although refers to cases of SSD<L, aims to associate the grade effect and the relevant SSD calculation directly with the K rate. From Equation (2) and Equation (4), it can be seen that the delivered sag vertical curvature rate is not grade dependent. The grade effect is indirectly introduced from the SSD determination and this explains the reason why the proposed process for delivering control sag vertical curve rates is based on the entering grade value. It must be stressed that regarding the AASHTO guidelines, 0 percent is assumed for the SSD calculation.

As already mentioned above, the authors have already assessed this issue for crest vertical curves, where revised crest vertical curvature rates were proposed, in order to grant SSD adequacy on exit areas of steep variable grades [6].

Table 1 illustrates the adopted design control values (rounded values) in the Green Book for SSD and sag vertical curvature rates respectively regarding various design speed values, based on the above equations. However, sag vertical curves shorter than the lengths shown in Table 1 may be justified for economic reasons in cases where an existing feature, such as a structure not ready for replacement, controls the vertical profile. Ramps as well maybe designed with shorter sag vertical curves. However fixed source lighting is desirable in such cases [1].

Tab.1 Design Control Values for SSD and Sag Vertical Curvature Rates

V_{design} (km/h)	SSD (m)	K (m)
50	65	13
60	85	18
70	105	23
80	130	30
90	160	38
100	185	45
110	220	55
120	250	63
130	285	73

In other design guidelines, although the same equations are utilized, the adoption of control sag vertical curvature rates is addressed through various considerations.

For example in the German RAA design guidelines for freeways [2] the SSD values used for the sag vertical curvature rate determination are reached for most unfavorable (negative) grade values.

The possible deficiency of the current SSD determination approach as adopted in the Green Book will be furthermore examined by introducing the grade effect during braking on sag vertical transitions.

3. Braking Calculation On Variable Grades

The current road design practices sufficiently address through Equation (1) the grade effect during the SSD procedure. However, the braking distance calculation for crest or sag curves that have variable grades is based on rather balanced assumptions. In most cases the algebraic mean grade value of the tangents preceding and succeeding the vertical curve is adopted. This concept though, fails to deliver the actual braking distances in all cases where the braking distance is less than the vertical curve length.

The work presented here regarding the evaluation of the effect of the variable grade during the braking process is based on a recently developed practice by the authors [5] briefly presented below.

Simple considerations based on the laws of mechanics through Equation (5) and Equation (6) were applied, assuming time steps of 0.01sec, in order to determine both the instantaneous vehicle speed and pure braking distance (SSD minus distance travelled during driver's perception-reaction time).

$$V_{i+1} = V_i - g\left(\frac{a}{g} + s\right)t \qquad (5)$$

$$BD_i = V_i t - \frac{1}{2} g\left(\frac{a}{g} + s\right)t^2 \qquad (6)$$

where :

V_i (m/sec) : vehicle speed at a specific station i

V_{i+1} (m/sec) : vehicle speed reduced by the deceleration rate for t = 0.01sec

t (sec) : time fragment (t = 0.01sec)

s (%/100) : road grade in i position [(+) upgrades, (-) downgrades]

BD_i (m) : pure braking distance

By applying Equations (5-6) subsequently there is a sequence value $i=k-1$ where V_k becomes equal to zero. The corresponding value of ΣBD_{k-1} represents the total vehicle pure braking distance for the initial value of vehicle speed. The variable grade SSD is produced by adding the final pure braking distance to the distance travelled during the driver's perception – reaction time [first component of Equation (1)] as follows:

$$SSD = V_o t_{pr} + \sum BD_{k-1} \qquad (7)$$

where :

V_o (m/sec) : vehicle initial speed

$\sum BD_{k-1}\ (m)$: total vehicle pure braking distance for the initial value of vehicle speed

Summarizing the SSD determination on variable grade values, the formula shown in Equation (1) is used, enriched by the actual grade value portions.

In terms of correlating the present SSD variable grade calculations procedure to the relevant found in the literature [10, 11, 12], it was found that the differences never exceeded ±0.70m. However the proposed approach is based on a selected time step (0.01sec) which compared to the average grade over the braking distance is more accurate.

4. Sag Vertical Curvature Rate Adequacy Investigation

The potential Green Book inadequacy regarding the suggested vertical curvature rates, must be sought in the negative grade area since on one hand downgrades increase the SSD for a vehicle and on the other, the current vertical curvature rate definition is extracted assuming flat vertical geometry.

In the following paradigm, an investigation regarding the sag vertical curvature rate sufficiency assuming 70km/h design speed is carried out, by defining the actual SSD values along two specified positions. Figure 1 illustrates the length of the consequent vertical curve adopted by the Green Book (K=23m), where the approach and exit grade values were set to -10% and +10% respectively. Two cases of vehicle braking are shown:

- Case 1, where the braking procedure begins at the starting point of the vertical curve (s=-10% where SSD=116.8m
- Case 2, where the braking procedure begins at the midpoint of the vertical curve (s=0% where SSD=99.5m)

In both cases the SSD definition was based on the calculation procedure described previously. As expected, the SSD value of Case 1 is greater not only from the relevant SSD value of Case 2, but from the SSD design control of 105m (Table 1) referring to 70km/h design speed as well. However, it must be stressed that Case 1 is not the most unfavourable case of excessive SSD value along the vertical curve. The most increased SSD value is delivered at the beginning of the curve and more specifically at the area where the pure (actual) braking procedure of the vehicle [second term of Equation (7)] commences, located 48.6m in advance of the sag vertical curve.

In general Figure 1 indicates that there are areas along the vertical curve where the braking procedure requires greater SSD values and thus an increase of the sag vertical curvature rate in these cases seems indispensable.

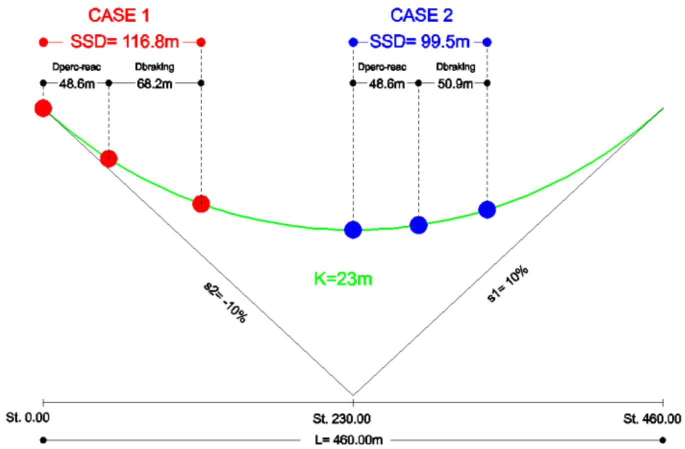

Fig.1 Discriminating Cases of SSD Variation on Sag Vertical Curvature Rate Suggested by AASHTO (V_{design}=70km/h, K=23m).

Figure 2 illustrates certain SSD values extracted for vehicle braking under 70km/h (design speed) against the control SSD of 105m. The corresponding control sag vertical curvature rate (K=23m) as well as the suggested (calculated) K values are introduced in the secondary vertical axis. Both the SSD and K values of Figure 2 were drawn arranging the pure braking procedure to be performed entirely inside the sag vertical curve. Therefore, the horizontal axis of Figure 2 exhibits the variable grade values within the sag vertical curve where the vehicle is supposed to begin the actual braking performance and is referred as entering grade value.

The data shows that at the beginning of the vertical curve, as the entering downgrade becomes steeper, the required SSDs increase accordingly which result to increased K values as well. For example, in Figure 2 when an entering grade of a sag vertical curve is set to -8% and the length of the variable grade area is above 117m, in order to grant SSD adequacy, the minimum sag vertical curvature rate value must be approximately K=26m, as opposed to the currently suggested K=23m. In Figure 1, by simple calculations the grade value where the pure braking procedure begins, or in other words, the instantaneous variable grade 48.6m after the entering grade of -10% is set to -7.87%. This grade value, according to Figure 1 delivers a SSD value of approximately 116.8m, which is confirmed by Figure 2. Commenting further on Figure 2, it can be seen that the assessment here, at least for the examined design speed of 70km/h, is not at all inconsiderable, since for the most unfavorable entering grade value (-12%), an increment rate of more than 25% is suggested for the sag vertical curvature rate.

In order the above procedure to be more integrated, Figure 3 through Figure 10 illustrate the suggested sag vertical curvature rates based on the vertical curve's entering grade value for design speed values of 50km/h, 60km/h, 80km/h, 90km/h, 100km/h, 110km/h, 120km/h and 130km/h. These figures assess the braking effect on steep (mostly) variable downgrades and thus deliver ready-to-use sag vertical curvature rate values for designers.

The entering grades utilized in Figures 2 through 10, were drawn based on grade control criteria as found in AASHTO's road functional classification. For example in Figure 3, the selection of grade values up to 14% for 50km/h design speed refers to mountainous local rural roads as recommended in the Green Book.

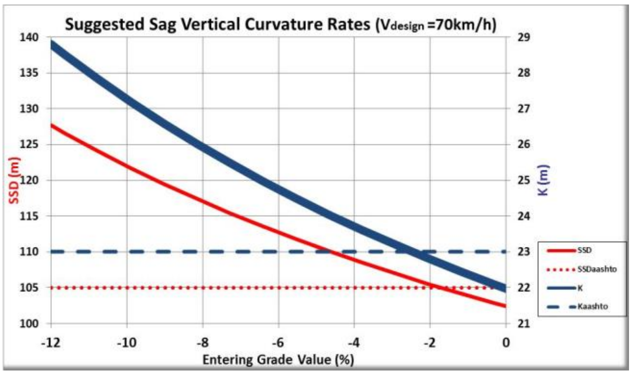

Fig.2 Suggested Sag Vertical Curvature Rate Values Based on SSD Adequacy (V_{design}=70km/h).

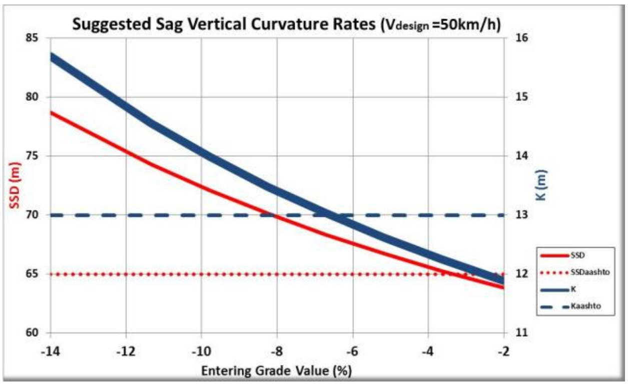

Fig.3 Suggested Sag Vertical Curvature Rate Values Based on SSD Adequacy (V_{design}=50km/h).

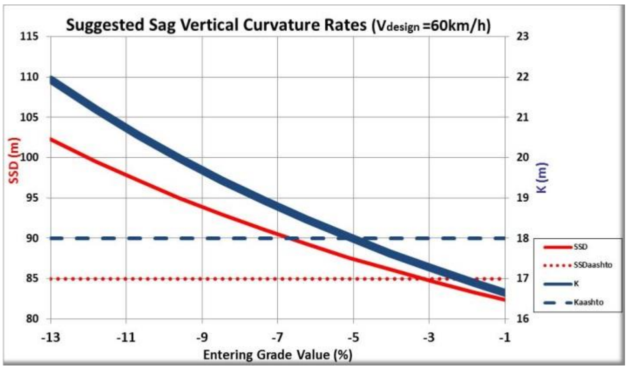

Fig.4 Suggested Sag Vertical Curvature Rate Values Based on SSD Adequacy (V_{design}=60km/h).

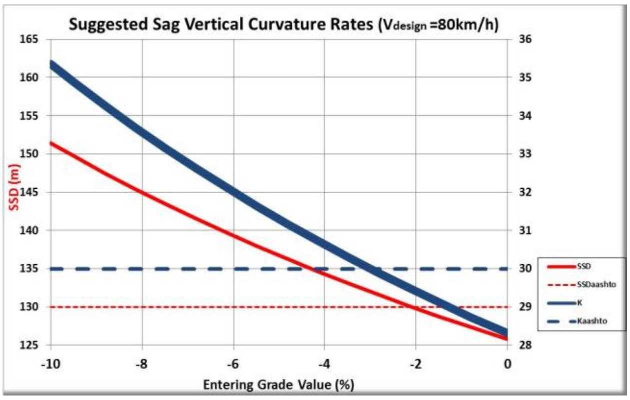

Fig.5 Suggested Sag Vertical Curvature Rate Values Based on SSD Adequacy (V_{design}=80km/h).

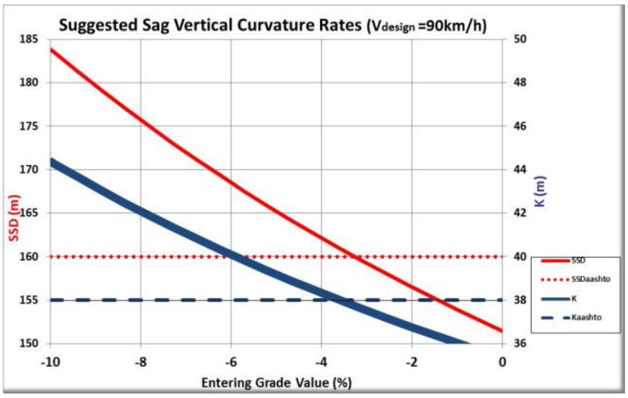

Fig.6 Suggested Sag Vertical Curvature Rate Values Based on SSD Adequacy (V_{design}=90km/h).

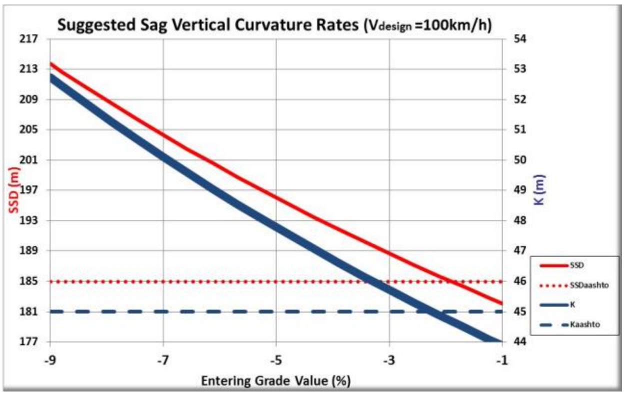

Fig.7 Suggested Sag Vertical Curvature Rate Values Based on SSD Adequacy (V_{design}=100km/h).

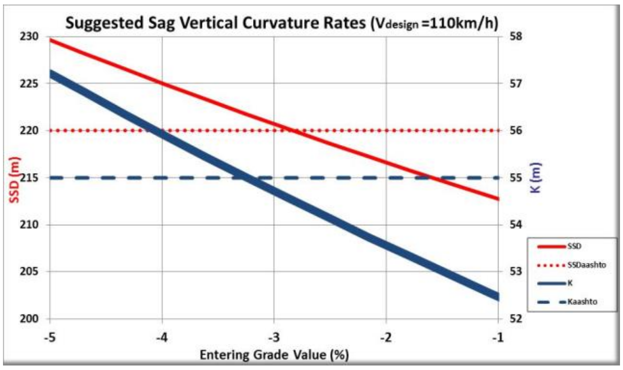

Fig.8 Suggested Sag Vertical Curvature Rate Values Based on SSD Adequacy (V_{design}=110km/h).

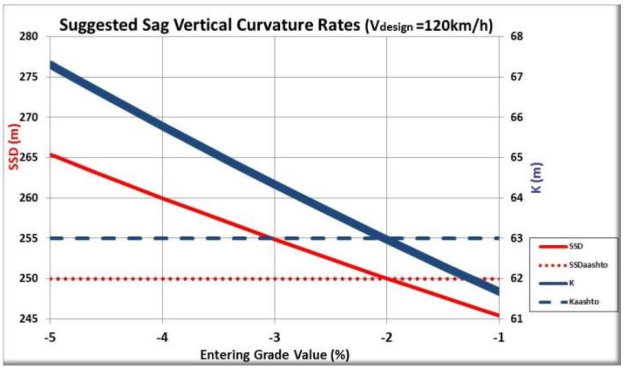

Fig.9 Suggested Sag Vertical Curvature Rate Values Based on SSD Adequacy (V_{design}=120km/h).

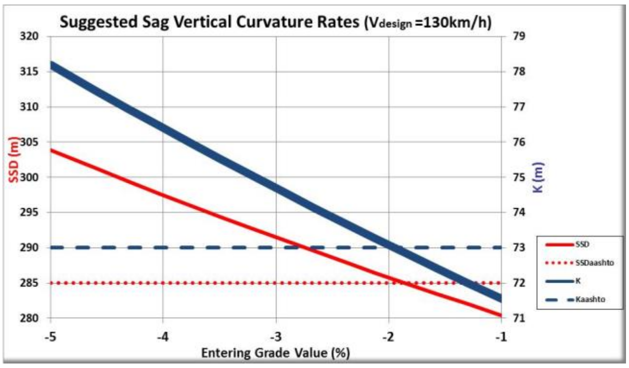

Fig.10 Suggested Sag Vertical Curvature Rate Values Based on SSD Adequacy (V_{design}=130km/h).

Figures 2-10 show that there is a horizontal variation between the SSDs' intersection point (red lines intersection) and the relevant K values intersection point (blue lines intersection). Theoretically these two intersection points should coincide horizontally, since both refer to the same ending grade value. However, this is not the case, since, the sag vertical curvature rates calculation formula based on Equation (2) and Equation (4), deliver values which are rounded for design. By closely examining the above figures, the following design speed values represent two extreme cases where the horizontal offset between the blue lines and the red lines intersection points is marginal and almost coincides respectively:

- V_{design}=50km/h, the K value is calculated 2.1m but introduced as 3.0m in [1]
- V_{design}=120m/h, the K value is calculated 62.8m but introduced as 63.0m in [1]

In the first case, the adopted K value for design is increased by 45%, where in the second only 0.3%. This explains the horizontal variation between the above stated intersection points.

5. Correlation Between Suggested Sag Vertical Curvature Rates Values and Road Functional Classification

The above analysis revealed a SSD inadequacy on the downgrade area of steep grades when control sag vertical curvature rates based on AASHTO design guidelines are utilized. On the other hand, as each road class is associated to a particular grade range, its impact in the above process is ignored.

The correlation between AASHTO's road functional classification and the amended sag vertical curvature rates as calculated here is shown in Table 2. It should be noted that this is illustrated for a single design speed of 80km/h for all possible functional classification categories.

Table 2, illustrates an example for ready-to-use values of sag vertical curvature rates, based on the desired entering grade value of the design. The illustrated sag curvature rates refer to negative entering grade values, extracted below a certain breakpoint where SSD inadequacy commences as shown in Figure 5, and can be applied in cases where the length of the vertical curve exceeds the required SSD values as shown.

The gaps in the suggested sag vertical curvature rates indicate that the maximum grade value of the certain road class is lower.

Tab.2 Suggested Sag Vertical Curvature Rate Values Based on Roadway's Functional Classification for V_{design}=80km/h

Note: L refers to length of vertical curve;

<table>
<tr><th rowspan="2"></th><th rowspan="2">Type of Terrain</th><th rowspan="2">AASHTO</th><th colspan="7">Entering Grade Value (%)</th></tr>
<tr><th>-4</th><th>-5</th><th>-6</th><th>-7</th><th>-8</th><th>-9</th><th>-10</th></tr>
<tr><td rowspan="3">Local Rural</td><td>Level</td><td rowspan="3">30m</td><td rowspan="3">31m (L>134m)</td><td rowspan="3">32m (L>136m)</td><td rowspan="3">32m (L>139m)</td><td>-</td><td>-</td><td rowspan="2">-</td><td rowspan="2">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">33m (L>142m)</td><td rowspan="2">34m (L>145m)</td></tr>
<tr><td>Mountainous</td><td>35m (L>147m)</td><td>36m (L>151m)</td></tr>
<tr><td rowspan="3">Rural Collectors</td><td>Level</td><td rowspan="3">30m</td><td rowspan="3">31m (L>134m))</td><td rowspan="3">32m (L>136m)</td><td rowspan="3">32m (L>139m)</td><td>-</td><td rowspan="2">-</td><td rowspan="2">-</td><td rowspan="3">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">33m (L>142m)</td></tr>
<tr><td>Mountainous</td><td>34m (L>145m)</td><td>35m (L>147m)</td></tr>
<tr><td rowspan="3">Rural Arterials</td><td>Level</td><td rowspan="3">30m</td><td rowspan="3">31m (L>134m)</td><td>-</td><td>-</td><td>-</td><td rowspan="3">-</td><td rowspan="3">-</td><td rowspan="3">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">32m (L>136m)</td><td>-</td><td>-</td></tr>
<tr><td>Mountainous</td><td>32m (L>139m)</td><td>33m (L>142m)</td></tr>
<tr><td rowspan="3">Urban Collectors</td><td>Level</td><td rowspan="3">30m</td><td rowspan="3">31m (L>134m)</td><td rowspan="3">32m (L>136m)</td><td rowspan="3">32m (L>139m)</td><td rowspan="3">33m (L>142m)</td><td>-</td><td rowspan="2">-</td><td rowspan="2">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">34m (L>145m)</td></tr>
<tr><td>Mountainous</td><td>35m (L>147m)</td><td>36m (L>151m)</td></tr>
<tr><td rowspan="3">Urban Arterials</td><td>Level</td><td rowspan="3">30m</td><td rowspan="3">31m (L>134m)</td><td rowspan="3">32m (L>136m)</td><td rowspan="3">32m (L>139m)</td><td>-</td><td rowspan="2">-</td><td rowspan="2">-</td><td rowspan="3">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">33m (L>142m)</td></tr>
<tr><td>Mountainous</td><td>34m (L>145m)</td><td>35m (L>147m)</td></tr>
<tr><td rowspan="3">Freeways</td><td>Level</td><td rowspan="3">30m</td><td rowspan="3">31m (L>134m)</td><td>-</td><td rowspan="2">-</td><td rowspan="3">-</td><td rowspan="3">-</td><td rowspan="3">-</td><td rowspan="3">-</td></tr>
<tr><td>Rolling</td><td rowspan="2">32m (L>136m)</td></tr>
<tr><td>Mountainous</td><td>32m (L>139m)</td></tr>
</table>

6. Conclusions

The paper investigates the consequence of the Green Book guidelines, to adopt minimum sag vertical curvature rates based on leveled grade values.

Initially, the authors addressed the SSD calculation on variable grades during the braking process through an earlier approach based on the point mass model and the laws of mechanics. This process resulted in determining that the negative grade area of sag vertical curves, as expected, increases the calculated SSD values.

Subsequently, for a wide range of design speed values, charts of the required SSDs were drawn as a function of various entering grade values, based on control sag vertical curve rates, as adopted by AASHTO. Since various SSD shortage areas appeared in the charts, the authors provided amended sag vertical curvature rates as well, in order to grant SSD adequacy throughout the braking process. The resultant sag vertical rates apply for cases where the length of the vertical curve exceeds the calculated SSD values. However, as stated in the Green Book [1], shorter sag vertical curves may be designed on the condition that fixed source lighting is provided.

An immediate implementation of the present approach is to provide the designers with ready-to-use revised sag vertical curvature rates, based on the desired entering grade value of the design in accordance to roadway's functional classification as adopted by AASHTO.

However further analysis is required in order to include the effect of combined horizontal – vertical alignment, certain arrangements of which might impose additional restrictions.

Moreover, additional qualitative research seems necessary to evaluate parameters of SSD (braking on curves, ABS braking, friction coefficient etc.) as well, in order to reflect current vehicle dynamics trends and thus simulate the braking procedure more realistically. One should not ignore the fact that the human factor might impose additional restrictions and consequently influence the braking process to some extent beyond the perception-reaction procedure and friction reserve utilized in the braking process.

Finally, it is also necessary to underline the fact that the parameters used in the present paper (speed values, perception reaction time etc.) refer to daylight driving conditions, as the vehicle speed values in night time driving conditions are 6km/h – 15km/h less [4] on one hand and on the other the road view geometry changes.

7. References

1. American Association of State Highway and Transportation Officials (2011). *A Policy on Geometric Design of Highways and Streets*, Fifth Edition. Washington, DC.
2. Ed.German Road and Transportation Research Association, Committee, Geometric Design Standards (2008). *Guidelines for the Design of Freeways, (RAA)*, Germany.
3. Hassan, Y., Easa, S.M. and Abd El Halim A.O. (1997). Design Considerations for Combined Highway alignments. *Journal of Transportation Engineering,* Vol 123, No.1, pp. 60-68.
4. Malakatas, K. (2012). *Operating Speed Predicting Model On Two-Lane Rural Roads During Night-Time.* Diploma Thesis, National Technical University of Athens.
5. Mavromatis S., Palaskas S. and Psarianos B. (2012). Continuous Three-Dimensional Stopping Sight Distance Control on Crest Vertical Curves. Paper published *on the Advances in Transportation Studies (ATS),* XXVIII issue, pp.17-30.
6. Mavromatis S., Stamatiadis N., Psarianos B., and Yannis G. (2015). "Controlling Crest Vertical Curvature Rates Based on Variable Grade Stopping Sight Distance Calculation". Paper *presented at the 94th Annual Meeting of the Transportation Research Board*, Washington DC.
7. Ministerio de Fomento (2000). *Instrucción de Carreteras, Norma 3.1 – IC "Trazado",* Spain.
8. Ministry of Environment, Regional Planning and Public Works (2001). *Guidelines for the Design of Road Projects, Part 3, Alignment (OMOE-X),* Greece.

9. Moreno Chou, A., Perez, V., Garcia, A. and Rojas M. (2010). Optimal 3-D Coordination to Maximize the Available Stopping Sight Distance in Two – Lane Roads. *Proceedings of the 4th International Symposium on Highway Geometric Design,* TRB, Valencia Spain.
10. Thomas, N.E., Hafeez, B., and Evans, A. (1998). Revised Design Parameters for Vertical Curves. *Journal of Transportation Engineering, American Society of Civil Engineers, ASCE,* Vol. 124(4), pp. 326-334.
11. Taignidis, I. and Kanellaidis, G. (2001). Required Stopping Sight Distance on Crest Vertical Curves. *Journal of Transportation Engineering, American Society of Civil Engineers, ASCE,* Vol. 127(4), pp. 275-282.
12. Hassan, Y. (2003). Improved Design of Vertical Curves Using Sight Distance Profiles. Transportation *Research Record 1851, Transportation Research Board,* Washington, D.C., pp. 13-24.