Multilevel analysis in road safety research

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Abstract

Hierarchical structures in road safety data are receiving increasing attention in the literature and methods are proposed for applying statistical models that appropriately handle hierarchical relations among the observations. These models are known as multilevel models. Road safety data may concern a broad range of response variables, including road accidents and casualties, exposure, safety performance indicators, behavioural indicators, and so on; these may be structured according to two types of hierarchies: geographical hierarchies (road safety data nested into road sites, nested into regions, nested into countries etc.), and hierarchies related to the accident process (i.e. road users nested into vehicles, nested into accidents). The former is often the case in macroscopic analyses, whereas the latter is typically to be considered in microscopic analyses.. This paper presents the theoretical background for multilevel analysis and discusses the objectives, model formulations, and underlying assumptions. A complete and comprehensive framework for multilevel analysis in road safety research is proposed. Moreover, the techniques are illustrated and assessed through a detailed presentation of numerous examples relevant to road safety research. As regards geographical dependences, the examples range from geographical multilevel analyses, to cross-country analyses, and to more sophisticated spatial models of road safety. As regards dependences in the accident process, the examples include analyses of the effects of various parameters on accident outcomes, mainly using in-depth accident investigation data. The review of the theory and applications of multilevel modelling in road safety research confirms the need for testing road safety data for hierarchical dependences, given that the consequences of ignoring such dependences may be important under certain conditions. On the other hand, a number of difficulties are identified in applying the proposed techniques on road safety data, particularly as regards the accident-process dependences. The balancing of the possible conceptual problems rising form not considering multiple levels in the analysis of road safety data on the one hand, and of the feasibility to apply the appropriate multilevel models on the other hand is a challenging question for further research.

Key-words: road safety; multilevel models; hierarchical structures; geographical dependences; road accident process dependences.

1. Introduction

Most of the data of interest for road safety research happen to be hierarchically organized, i.e., to belong to structures with several hierarchically ordered levels. The observable units can be defined within each level in a way that each unit on a lower level can unambiguously be assigned to one and only one unit on the higher level. These hierachical structures result for a part from the *spatial (and temporal) distribution* of the various data collected, for another part from the very nature of *accidents.* From a spatial point of view, the observations made belong to larger geographical areas or units (these can be as various as road sites, segments, or intersections, counties, or regions). In a similar vein, data concerning vehicles, drivers, of individual road users involved in accidents are inevitably “clustered” within the accidents: the observations are derived from higher-level units: the accidents, as each road-user, driver or vehicle observation “pertains” to one and only one accident.

One of main problem associated with hierarchical data organisation is the *dependence* that they generate among the observations. Observations that are sampled from the same geographical higher-level units have in common a series of unobserved characteristics that are proper to these larger geographical areas, which are heterogeneous themselves. One can think of road-side surveys where observations about the behaviour of drivers are made from various road sites, some situated in the vicinity of cafés and restaurants (drink-driving), others being remote from them, some situated in highly congested areas, others in areas where the flow of traffic is much more free (speed). One can also think of intervention studies that are based on crash-frequency data aggregated over a sample of road intersections or segments, … Whenever accident data are disaggregated at the individual level (drivers or all individuals involved in accidents, for example), account should be taken of the fact that observations made on individuals occupying the same vehicles and involved in the same accident are likely to resemble each other more than observations made on individuals involved in different vehicles or accidents. This is so because these observations will be commonly influenced by vehicle and accident characteristics of that are often left unobserved in a given analysis.

The estimations obtained from most standard analysis techniques rest the assumption that the observations are sampled from a single homogeneous population, and that the residuals resulting from the model are independent. However, the hierarchical organisation of data fundamentally challenges these assumptions. *Applying traditional statistical techniques (linear of generalized linear models) results in underestimated standard errors and exaggerately narrow confidence intervals. “This is especially the case when the risk factor corresponds to a crash or car feature “collapsed to the level of the casualties and simply replicated across all individual sharing those characteristics” (Lenguerrand et al., p44)*

Statistical models have been developed that allow accounting for hierarchical data structures, and consequently taking the dependency they introduce among the data into account. These are the so-called multilevel or hierarchical models. This starts with a definition of hierarchical models and of some theoretical concepts that fundamentally relate to them. We also briefly discuss the advantages of hierarchical data analyses compared to other statistical techniques that have been applied to address the issue of dependency in data. We then provide a comprehensive overview of the hierarchies that are most often encountered in road safety research, we discuss the application of hierarchical model taking into account the types of dependent variables that are frequently measured at the different levels of the hierarchy, along with the associated response distributions.

Definition and general model formulation:

The essential feature of hierarchical models is the fact that the model specifies the observations at the lower levels as being clustered into higher-level units, and that these units themselves are considered to constitute a sample from a larger population (of accidents, of road sites, or segments, …). A multilevel/hierarchical model can thus be defined as a regression model (based on linear or generalized linear models) in which the regression coefficients are assigned a probability model. The higher levels of the model have parameters of their own – the "hyperparameters" of the model – which are also estimated from the data (Huang & Abdel-Aty, 2010, p. 1560).
Or: ML/hierarchical model are defined as model in which the parameter are allowed to vary across units situated at higher levels of the hierarchy.
Hierarchical models are models in which the effects of higher levels on the parameters (intercept and covariate coefficients) estimated on the basis of observations made at a lower level.

To further define the principles underlying ML models, a simplified two-level model will be used first. The response variable corresponds to the probability for driver *i* involved in accident *j* to die as a result of this accident.

The response y_{ij} is defined as being a function of the expected value defined for accident *j* (π_j) and of driver-specific variation (R_{ij}).

$$y_{ij} = \pi_j + R_{ij} \qquad (2.3.5)$$

The expected average probability for each accident is in turn defined as being a nonlinear function of a linear combination of predictors.

$$\pi_j = f\left(X\beta_{ij}\right) \qquad (2.3.6)$$

where f(x) represents the link-function, the logit link being chosen for this example[1].

$$Logit\ (\pi_{ij}) = \gamma_0 + \sum_{h=1}^{r} \gamma_h x_{hij} + u_{oj} \qquad (2.3.9b)$$

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The expected probability for driver *I* to die in accident *j* is now defined as being a logit function of the linear combination of an average value holding for the accident population (γ_0), of the effect of level-1 (and/or) level-2 predictors ($\sum_{h=1}^{r}\gamma_h x_{hij}$) and of accident-related random deviation u_{oj}.

$$Logit\,(\pi_j) = \gamma_0 + u_{0j} \qquad (2.3.7)$$

where γ_0 represents the average of logit(π_j) across accidents and u_{0j} the accident-specific deviation from this population average value (the "accident-level random variation). These deviations are assumed to be normally distributed, with mean 0 and variance $\sigma^2_{u_0}$

The model just defined allows the model intercept to vary on the basis of the accident, so that the expected probability of dying can be higher for some accidents than for the others. This variation is meant to represent the influence exerted by the unobserved characteristics of the accident-level units on the observations made on the individual drivers, and thus to account for the dependence introduced at that level.

The technique moreover allows the specification of accident-level predictors at the correct level of analysis (predictors that are assigned the subscript…). Had this higher level been ignored, while accident-level predictors were considered of being of interest for this analysis, the only solution one would have had would be to replicate every accident-variable value across for all the drivers involved in the accident, and to consider them as "independent driver observations".

Models such as the one presented above are called random intercept or variance components models. The only parameter that is allowed to vary across higher-level units is the intercept. These models allow quantifying the part of the random variation in the observations that is associated with with the higher level considered (so, the proportion of variance in the outcome: the probability for drivers to die as a result of the crash that results from between-accident variation).

Indeed, the outcome variance is now partitioned into two components: the variance of the u_{0j} and the variance of R_{ij}. The intraclass correlation coefficient establishes the ratio of the higher level variation to the total variation in the observations. However, in the case where discrete responses are modelled, the variance of R_{ij} is not available (the model in equation xxx does not contain a parameter for the level-1 variance). The reader is referred to Goldstein (xxxx) and Bryk and Raudenbush (xxx) for methods to appropriately estimate the ICC for non-normally distributed responses.

In a next step, the coefficients for the covariates included in the model ($\sum_{h=1}^{r}\gamma_h x_{hij}$) can also be defined as varying randomly across the higher-level units. The accident-level random variation of the covariates ($u_{1j}x_{1ij}$) is then added to the model which is now written as:

$$Logit\ (\pi_{ij}) = \gamma_0 + \sum_{h=1}^{r} \gamma_h x_{hij} + u_{oj} + u_{1j} x_{1ij} \qquad (2.3.10)$$

Observing that the effect of driver characteristics, age for example, can be considered uniform (fixed) across all accidents. A significant variation of this predictor's effect at the accident level would suggest an interaction between some unobserved accident characteristic (the type of impact, for example) interacts with the driver-level characteristic (age) to determine the probability of the driver to die or survive the accident.

All residuals components defined at the higher level higher level (μ_{0j} and μ_{1j}) are assumed to be:

(1) normally distributed with mean 0 and variance $\sigma^2{}_{u_0}$ and $\sigma^2{}_{u_m}$;
(2) independent across the j-units
(3) independent from the residuals at other levels of the model

The higher-level random terms estimated on the basis of ML models reflect the correlations between the observations that are induced by the unobserved characteristics of the higher-level units. In the case of a correctly specified ML model, the residuals at the lowest level can be said to be independent, *conditional on other effects in the model*. In other words, the error term at the lowest level is "cleaned" for the influences of the higher levels by the specification of the corresponding random effects, and can thus be considered to display independence. .

3.2. The importance of using multilevel models in road safety:

The use of hierarchical models improves the correctness of the estimation and inferences made from hierarchically structured data. It is thus warranted and recommended for purely statistical reasons.

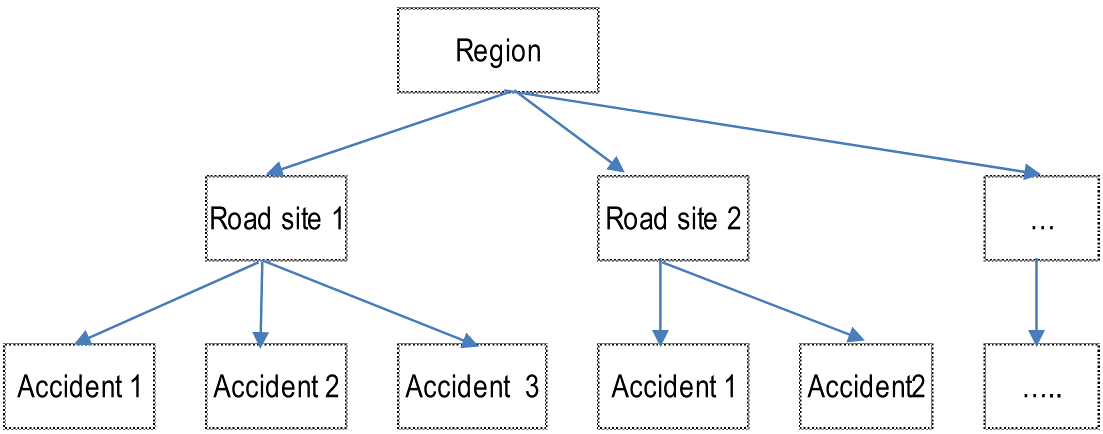
Apart from that, it is important to recognize that, in comparison to traditional, models, the specification of ML models forces the researcher to refine his/her view of the phenomenon investigated. These models require that attention is paid to the level at which the observation units are defined, as well as to the level(s) at which the predictors of interests are expected to exert their influence. Actually, many problems in road safety research cannot be understood correctly if only one level is considered, and interpreting results without taking the hierarchical framework into account can lead to erroneous conclusion. The error that is frequently made consists of considering that the relationships that are observed at given levels of the hierarchies also hold for the others.

This is nicely illustrated by a simulation study conducted by Davis on the probability of car-pedestrian collisions (2002, quoted in Davis (2004)),. On the basis of actual observations, vehicle/pedestrian encounters were simulated, and the relation between collision probability and variables such car speed or the traffic volume was assessed. The data simulated corresponded to three levels of aggregation: the individual pedestrian/vehicle encounter, the road site level, and the “population level”, combining the information from all road sites. On the basis of the population-level aggregate data, it appeared that collision probability was clearly related to traffic volume, but not to speed. This suggests that speed is unimportant for pedestrian safety. The analyses made at the road site level, however, showed that the value of the slope for the car speed–pedestrian risk relation appeared to depend on traffic volume, so that at certain road-sites no relation could be observed between speed and collision probability, while a positive relation was observed between both variables at other road sites. Working at the road-site level allowed holding that characteristic constant, and thus allowed for the relation between the 2 predictors at the lower level to show up. The term “ecological fallacy” is usually used to refer to instances where inferences made at one level of analysis are erroneously and straightforwardly applied to other levels.

2. A general hierarchical framework for road safety

2.1. Prevailing hierarchies in road safety research: Spatial distributions of data and the nature of the accident process

One can distinguish two prevailing hierarchies in road safety research data, namely: geographical and accident hierarchies.



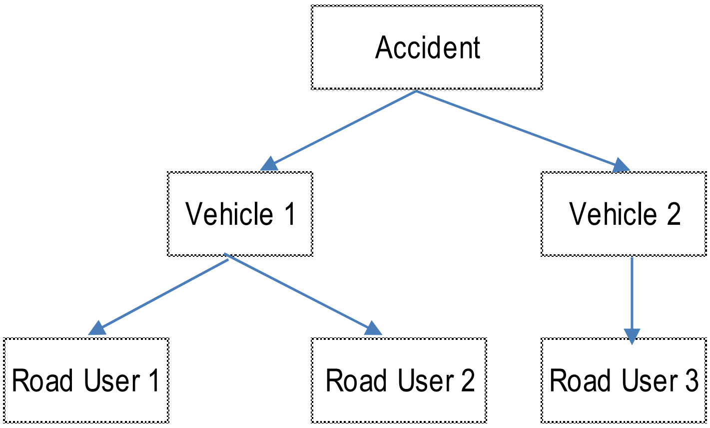


Figure 1. Geographical (top panel) and accident (bottom panel) hierarchies of road safety outcomes

As illustrated in Figure 1, road safety data are organized in geographical units that are nested into each other (for example: road-sites nested into counties that are themselves nested into regions and countries). Similarly, the observations made on individual road-users are nested into vehicles, which are themselves clustered into different accidents.

The two hierarchies are actually complementary and have been incorporated into a single framework to represent prevailing data structures in road safety (Huang & Abdel-Aty, 2010). In Figure 3, an adapted version of this general hierarchical framework.

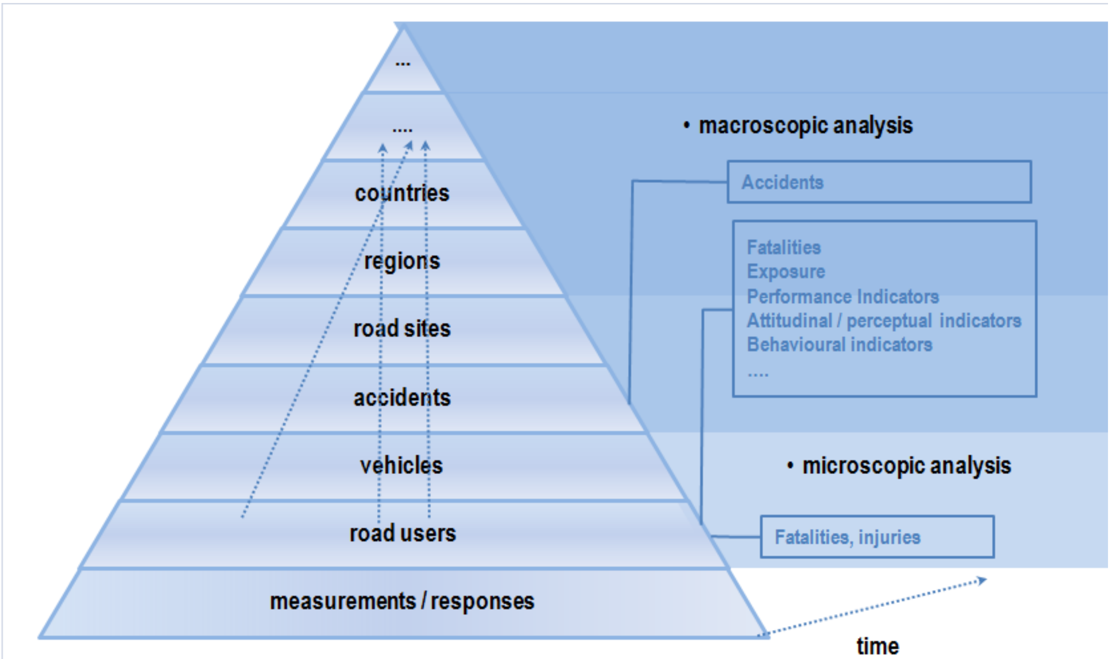


Figure 3. Framework for multilevel analysis in road safety research (adapted from Huang &Abdel-Aty, 2010)

This hierarchy can be described as follows:
The first "geographical" level represents the various types of entities (countries, regions, states…) within which lower-level observations can be

nested. “This level is normally associated with a number of contextual factors potentially affecting the traffic safety situation, such as driving regulations, road density, spatial features, population and other socio-economic features” (Huang & Abdel-Aty, 2010, p. 1559).

The next level, ‘road-sites” designates entities defined to operationalise the data collection (sampling) of various types of data on various locations defined in a homogeneous way (road segments or road junctions, for example). As such, they play an important role in accident frequency studies, but also in interventions studies (when, for example, the efficiency of some treatment is evaluated by pairing and comparing road sites with the treatment applied to others where the treatment hasn’t been applied). Road sites are also used for the collection of behavioural data in road-side surveys.

The accident hierarchy is rather self-explaining: road-users are nested in vehicles which are themselves nested in accidents. It is important to note, following Huang and Abdel-Aty (2010) that, depending on the specific research question and of the particular types of observation made, “drivers” can be considered as part of the vehicle level or of the individual road-users level. Severity studies based on driver injury severity and/or vehicle damage (example) are an example of cases where drivers and vehicles could be defined as “exchangeable units”. Studies focusing on accident causes, where the behaviour of driver is a “characteristic” common to all other vehicle occupant are another example. When injury severity of injuries – or the survival probability – of the individual road users is the focus of the research, however,

Drivers and driver-related covariates can be defined either at the vehicle or “road-user” level depending on the focus of the study. When injury severity or vehicle crashworthiness is concerned, drivers should be considered as “road users” like others. The driver characteristics included in the model should, however, be carefully assigned to the appropriate level: covariates indicative of the drivers’ *behaviour* are common to all road users in the same vehicle and should consequently be defined at the vehicle level. The personal characteristics of the drivers that are likely to affect their vulnerability should, however, be treated just like the characteristics of interest for the other vehicle occupants and defined at the “road-user” level.

In Figure 3 a lower level is foreseen “level 0”, to express the capacity of multilevel of handling complex types of response variables (multivariate responses, multinomial responses, or repeated measurements) as being nested within individuals.

A horizontal ‘time’ dimension can be also included in the framework (Huang & Abdel-Aty, 2010; Aguero-Valverde & Jovanis, 2006), as time is another dimension susceptible to create dependences in the observations (two observations that are made closer in time will resemble each other more than two observations that are made further apart in time).

- The multilevel structure can also be a multiple membership structure, as indicated by the double arrow inside the pyramid, or a cross-classification structure, as indicated by the crossed arrows inside the pyramid.

Intuitively, geographical hierarchies call for macroscopic analysis, while accident hierarchies, with individual road users or drivers as unit of analysis are the ideal basis for microscopic analysis (e.g.: what are the accident, vehicle, or driver characteristics that help predicting the occurrence of accidents and/or their outcomes?).

For the sake of clarity, we will use the term "aggregate analyses" to refer to studies in which the response variable – indicators of accident frequency in most cases – is aggregated over some geographically defined units (road sites or intersections, for example) and to "disaggregate analyses" whenever the response variable is defined at the accident level (for example: the most severe injury sustained by the road users involved in the same accident), at the vehicle level (the severity of the drivers' injuries), or at the road-user level (the severity of the injuries sustained by each and every road-user involved). Studies focusing on the behaviour of individual drivers, or on the attitudes of individual road-users constitute a special case, in the sense that the response variable is measured at the lowest levels of the hierarchy presented in Figure 3. However, the intermediate levels defining the accident hierarchies are irrelevant for understanding and correctly specifying the variation of the observations. The highest levels (road-sites, geographical areas) are nevertheless likely to be of importance.

In the following section, the usefulness of ML modeling is illustrated on the basis of empirical examples. We first discuss studies focusing on accident frequency and the geographical distribution of the accident data. Then, disaggregated analyses of accident severity are reviewed, and finally studies aiming at collecting information on driver and road-user behaviour are discussed. In each case, the type of distribution followed by the response variables is defined, the level at which the observations are made along with the levels that are explicitly accounted for in the models are indicated.

Aggregate analyses of accident frequency

The analysis of aggregated data, focusing almost exclusively on the geographical part of the hierarchy can be generally considered to fall within the broader family of spatial analyses.

These may range from ecological analyses, examining road accidents or fatalities nested within sites, areas or regions (Yannis et al. 2007), to complex spatio-temporal analyses (Aguero-Valverde & Jovanis, 2006; Eksler, 2008). This type of hierarchy is conceptually intuitive and computationally undemanding in most cases, and can be considered to be the most broadly used multilevel framework for road safety analysis. Obviously, the number of geographical levels depends on the scope of each research question and the hierarchy can be much more detailed than the broad 'accidents into road sites into regions'. Sometimes, the observations belong to several geographical units at the same time (multiple memberships), or need to be classified into two distinct but non strictly hierarchical dimensions at the same time (cross-classifications).

The simplest way to account for spatial dependence on the basis of ML modeling is to define (more or less arbitrarily) higher geographical units which are assumed to differ from each other on a number of characteristics (…), and within which the units of observations are clustered.

Spatial dependence is in this case refers to a general co-variation of properties within a geographical space. One way to account for spatial dependence is by specifying the hierarchical organization of the observations in the statistical model, as it is done when modeling “road-site accident counts nested into regions”, for example.

When applied to road safety, the basic idea behind spatial modelling techniques is the decomposition of the random variation of road accident risk into two distinct components: a "structured" component, which is assumed to represent the spatial structure of the road safety outcomes, and an "unstructured" component, which is assumed to be random. Therefore, the road safety outcome Y_i of county (i) is considered to be the result of systematic variables effects $\beta_i x_i$, and random variation ε_i, which is further decomposed into "structured" random variation (u_i) and unstructured random variation (v_i). Typically, a Poisson distribution is assumed for road safety outcomes, with an exposure estimate (e.g. the population N_i) incorporated as an offset term.

Spatial models are based on a "neighbourhood matrix", in which the spatial structure is defined through the identification of neighbouring geographical units. Obviously, the neighbourhood matrix needs to be symmetrical. The way this neighbourhood matrix is used differs for the various methods. A detailed presentation of the formulation and statistical properties is beyond the scope of this article, and only a summary of basic assumptions is presented here. The reader is referred to Eksler and Lassarre (2008) for a complete presentation.

The MMMC model is a multilevel model, which combines a cross-nested structure and a multiple membership structure. The cross-nested structure is used to describe the fact that the variation in the observations comes for a part from the geographical unit they belong to (i.e. overdispersion in the accident counts), and for another part from the neighbourhood structure (i.e. spatial effects). The multiple membership structure is used to describe the fact that each region has more than one neighbour. The MMMC model assumes that geographical units are separate entities. Two types of random effects are estimated in an MMMC model: an "exchangeable geographical unit effect" to account for overdispersion, and, to account for spatial dependence, (r) random effects for each geographical unit, with (r) equal to the number of its neighbours. The remaining random variation is the unexplained part of the model.

The CAR model uses a slightly different approach, in which geographical units are no longer considered as separate entities (Browne, 2004) and a conditional auto-regressive distribution is initially assumed for the structured variation. Consequently, in the CAR model, the set of neighbours of each geographical unit is initially examined as a whole, and one (global) random neighbourhood effect is estimated for each observation. Moreover, no exchangeable random effect (overdispersion) is initially assumed in the CAR model. Incorporating overdispersion gives the CAR convolution model, in which both the structured and the unstructured components are estimated, through a mixture of exchangeable normal and conditional autoregressive distributions.

In general, the CAR method results are considered to be more reliable, for both theoretical and practical reasons.

In a Greek study on the effects of speed and alcohol enforcement on road safety (Yannis et al., 2007), fatal accident and fatalities, as well as the number of alcohol controls and of speeding tickets for a period of 5 years were registered for each county. The data were analysed in multilevel poisson-family regression models, including poisson, extra-poisson and negative binomial models, with counties-aggregated accidents counts nested into regions. It turned out that both enforcement measures were highly correlated (i.e. in counties where the police executed many alcohol controls were also those in which many speeding tickets were issued), and that they together led to a significant decrease in fatalities. Moreover, there was significant regional variation in the number of accidents and in the related effect of enforcement. In particular, the enforcement measures proved most effective in those regions that had the highest accident rate in the first place. The same data set was analysed in a multivariate multilevel model (Yannis et al. 2008), allowing the simultaneous investigation of the effects of enforcement measures on two road safety outcomes (fatality and accident counts). The two outcomes were correlated, and part of their covariance was situated at the regional level. As was the case in the previous analyses, the regional variation of the effect of enforcement was significant for accident counts, but not for fatalities. A possible interpretation is that enforcement has an important overall effect on fatal accidents because these result from more risky behaviours. The reason why the enforcement effect on fatalities is uniform for all regions would be that drivers perceived an increased nationwide presence of the police and improved their overall behaviour accordingly. The decrease of non-fatal accidents (which result from less risky behaviours) would, however, have reflected differences in local enforcement practices and have been more or less important from one region to the other as a consequence.

In a study aiming at examining the relation between the safety of road segments in Vancouver to their traffic and geometric characteristics, El-Basyouny and Sayed (2009) took into account the clustering of the road segments into larger geographical units or “corridors”. They fit both random parameters and random coefficients models. The results indicate that a substantial part of the variation in accident counts at the road-segments level is attributable to the “corridor level” (random intercept) and that the effect of several of the road-segment characteristics investigated also varies significantly from one corridor to the next. Moreover, some of the parameters estimated, while being significant when estimated on the basis of a standard regression model are not significant any more once the hierarchical organization of road segments into corridors is properly accounted for.

Papadimitriou et al. 2011 applied a neighborhood matrix to the analyses of Greek-counties accident counts described earlier. The neighborhood matrix was defined to account for the road connections between counties. The results suggested that the spatial structure examined accounts for an important part of the variation in road accidents in the Greek counties, revealing a general pattern of risk increase from northern to southern Greece. Importantly, the results indicated that the effect of enforcement would have been quite overestimated had spatial effects not been taken into account.

Guo et al. (2009) modeled the spatial dependence among road-site accident counts by specifying the clustering of road sites into larger corridors[2] and examining the variation of the model coefficients at the corridor level. These authors also fitted a Conditional AutoRegressive (CAR) model to account for the spatial dependency in a refined way, by modeling the "micro-level spatial correlation". Whatever the type of model used, the authors observed that several parameters became nonsignificant once spatial dependence is accounted for and that "ignoring overdispersion and spatial correlations among observed data lead to overly optimistic and invalid statistical inference" (p. 91).

Disaggregate analyses of accident severity

The analysis of disaggregated accident data, on the other hand, focusing on individual road safety casualties, requires the accident hierarchy to be taken into account. Of course, road users or accidents can be further nested into road sites, areas, regions and so on, expanding thus the hierarchy towards geographical elements. Because road sites can be considered to belong to both hierarchies, they constitute the link between geographical and accident hierarchies, the macro- and microscopic ML structures. Consequently, although geographical and accident hierarchies have different scopes and properties, they are also inter-related, so that one might consider that the accident hierarchy as a whole is nested into the geographical hierarchy. Road sites can consequently be seen as the elements that "link" the macro- and micro- levels of the hierarchies.

An important specificity of accident-like hierarchies deserves to be briefly discussed at this point: while the number of accidents is usually rather large, the number of cars per accident and of individuals per car is typically very low. Theoretically speaking, it makes sense to consider that outcomes will be affected by unobserved accident and vehicle characteristics, and that

[2] "Corridors" are in this case explicitly defined "as a multi-lane highway with high speed limit and serving relatively long trips between major points" (Guo et al., 2009, p. 85).

observations made in the same vehicles and in the same accidents will consequently be correlated. The low numbers of sub-units at each level can however cause computational difficulties when estimating the random parameters in the model. On the basis of the analysis of observed accident data, and of 100 simulated datasets with unit-per-cluster numbers that are representative of those generally observed in accidents (i.e., less than 2 cars per accident and less than two occupants per vehicle), Lenguerrand et al. (2006) observed that the estimation of the variance of the random effect is problematic: the variance of the vehicle random effects is falsely estimated as "0" in 36% of the cases, and the standard deviation of the random effects' variance was often incorrectly estimated as 0 as well). These incorrect estimations are clearly related to the small number of observations available per vehicle and per accident[3]. The ML formulation, however, provided the less biased estimates for the fixed effects also included in the model. According to the authors, this is due to the fact that the covariates estimates are adjusted to other confounding factors related to the occupant-vehicle-accident structure. The authors conclude that ML models are best applied – in the context of disaggregate accident data – to large accident datasets, accident data characterized by large numbers of occupants per vehicle, or when the results obtained from traditional models seem questionable (in contradiction with earlier empirical findings, for example).

Two studies, focusing on the outcomes of fatal accidents for the individual road users involved. As in many "severity studies", the aim was to investigate the relationship between various accident, vehicle-driver and road-users characteristics to the probability for each road-user to sustain injuries of different severity levels (modeled as a multinomial response, Papadimitriou et al., xxx) or to survive the accident (modeled as a binomial response; Dupont et al., 2011). On the basis of the first study, no random variation at the vehicle and accident level, with the exception of the probability of serious and slight injury which appeared to vary significantly across vehicles. In the second study, no significant random variation was observed, either at the accident or vehicle level.

Often, this problem is solved by "ignoring" the vehicle level, and by directly modeling the nesting of individuals within accidents instead (Lenguerrand et al., 2006; Jones & Jorgensen, 2003). Other studies have selected the "vehicle-driver entity" as observation unit, so that the lower level, namely, that of occupants within vehicles, is ignored (Huang, Chin, & Haque, 2008).

Behavioural studies:

Data from a Belgium roadside survey on seatbelt use were analysed in a single-level and a multilevel framework (Vanlaar, 2005a). These data were collected at randomly selected road sites: Seatbelt wearing was recorded for the driver and, if present, the front passenger of each car passing on the road-site. The probability of seatbelt use was modeled by means of a binary logistic regression model (yes / no). The results showed significant variation between

[3] The correctness of the estimates of the variances of the random parameters increases when the number of observations per car or accident units increases in the simulated data.

road sites in the probability of wearing a seatbelt, suggesting that the ML approach was appropriate. The speed limit at the road site could explain some of this variation (drivers on roads with higher speed limit had a higher probability of wearing a seatbelt), but not all of it. There was of course also significant variation between the drivers themselves, some of which could be explained by gender i.e. women tend to wear seatbelts more often than men (Vanlaar, 2005a).

Results obtained on the basis of a single and multi-level approach were also compared for another road-side survey conducted in Belgian, this time focusing on drink-driving. The result of the alcohol checks is expressed in Belgium on the basis of 3 values, indicating whether BAC (breath alcohol concentration) is below 0.05 mg per litre (the legal limit), between 0.05 and 0.08 mg, or above. A first way to analyse these data is to dichotomize them by merging the two upper categories and simply differentiating between blood-alcohol concentrations under or above the legal limit (i.e. by means of a binary response variable) (Vanlaar 2005b). The original three response categories can also be kept in the analysis, and model as an ordered multinomial response (Dupont & Martensen, 2007). Among predictors defined at the road site level, the time of testing was the most important predictor, as the probability of drink driving on weekend nights by far exceeds that at all other time points. At the individual level gender and age were the most notable predictors with, men between 40 and 54 having the highest risk of drink driving.

- A road safety question is associated either with individuals or with accidents, in the form of a response variable of different types.

Especially as regards the response variable, different road safety questions may be examined, resulting in different response variables. More specifically, in accident hierarchies, the response variable will be the number of casualties (fatalities or injuries). In geographical or practical hierarchies, however, a broad range of response variables may be considered, including:
- Road accidents or casualties i.e. fatalities, injuries etc.
- Behavioural or performance indicators, such as the driver's speed, the BAC level, the use of seat belt or helmet etc.
- Attitudinal or perceptual indicators, such as the acceptance of road safety measures, the perceived risk of speeding or drink-driving.

Obviously, these different types of road safety questions will be typically attached at the road user level (or at the accident level, if accident counts are examined), and expressed on the basis of response variables that may be normal, discrete (binomial, multinomial, count), multivariate etc.

Accident analysis, risk analysis, severity analysis behavioural analysis

4.5. A note on estimation methods for multilevel models

For most of the models presented in the previous sections, conventional default estimation methods can be used in the modelling process. The default estimation methods are either maximum likelihood or some approximation of maximum likelihood (e.g. quasi-likelihood), which are based on Generalized Least Squares estimation (GLS) (Dupont & Martensen, 2007; Browne et al. 2001).

However, an important problem often rises from the use of approximation methods, as the estimated likelihood ratio is very approximate and cannot be used for the assessment of fit of the model. Moreover, when these methods are applied to more complex data structures, such as the "non-purely hierarchical" structures mentioned above, numerical and convergence difficulties are encountered.

A group of alternative estimation methods for multilevel models, based on Bayesian inference (e.g. the Markov Chain Monte Carlo (MCMC) and the bootstrap methods) may be used. These advanced methods are both based on simulation techniques and the estimates they produce are dependent on randomly generated numbers. In contrast to the conventional methods, where an estimate for a parameter (mean and variance) is obtained on the basis of a single sample, these simulation methods generate a large number of samples from the initial sample, providing thus a sample of parameters (means, variances), and allow for the calculation of intervals of parameter estimates. Because they are based on intervals estimates, they also allow for the calculation of accurate likelihood values (Dupont & Martensen, 2007). These simulation-based estimation methods are also more powerful in dealing with complex data structures, as well as with datasets with missing data, or with few data (Huang & Abdel-Aty, 2010).

Case study	Response variable and values	Response variable type	Statistical method	Analysis		Levels*					Random effects
				microscopic	macroscopic	Vehicle	Accident	Road site	Region	Country	
Drink-driving survey in Belgium	Seat belt use (yes/no)	Discrete, Binary	Logistic regression		•			•			Significant
Drink-driving survey in Belgium	BAC (<0.5gr/lt, >0.5gr/lt)	Discrete, Binary	Logistic regression		•			•			Significant
Drink-driving survey in Belgium	BAC (<0.5gr/lt, 0.5-0.8gr/lt, >0.8 gr/lt)	Discrete, Multinomial	Logistic regression		•			•			Significant
Road safety enforcement in Greece	Accidents	Discrete, Counts	Poisson Quasi-Poisson, Negative Binomial regression		•				•		Significant
Road safety enforcement in Greece	Accidents, Fatalities	Discrete, Counts	Poisson regression		•				•		Significant
Spatial road safety analysis in Greece	Accidents	Discrete, Counts	CAR model, Multiple membership (MM) model		•				•		Marginally significant (MM) or significant (CAR)
Joint analysis of CARE and SARTRE data	Accidents, Fatalities	Discrete, Counts	Quasi-Poisson regression		•					•	Significant
Injury severity mis-reporting	Police and Safetynet severity scores (lower score, matching, higher score)	Discrete, Multinomial	Logistic regression	•		•	•			•	Non significant
Injury severity in fatal accidents (all road users)	Injury severity (fatality, serious or slight injury)	Discrete, Multinomial	Logistic regression	•		•	•			•	Significant (vehicle)
Injury severity in fatal accidents (car occupants and their opponents)	Injury severity (fatality, non fatality)	Discrete, Binary	Logistic regression	•		•	•			•	Non singificant

*Levels designated by a red dot indicate the level at which the response variable is operationalised.

6. Discussion

6.1. When is it necessary to use multilevel modeling?

The results described above confirm that hierarchical dependences are very often encountered in road safety problems, and that a multilevel representation may offer a solid conceptual and computational basis for the analysis, both when it comes to geographical hierarchies and accident hierarchies.

The question remains, however, of knowing whether the hierarchy-induced dependences in the observations cannot be accounted for on the basis of other, more simple methods. Choosing to focus on drivers, for example, is a simple way of performing an individual severity analysis while avoiding vehicle-induced correlations. Yet, this selection comes at the cost of data and information losses, and it also does not prevent the researcher from committing the “ecological fallacy” when straightforwardly concluding that driver-based observations hold for other vehicle occupants.

One could also argue that dependences in the observations can be corrected for by directly taking up higher order variables (e.g., the speed limit or traffic limit at the road site), without actually introducing a higher level (road site itself) into the analysis. Such an assertion amounts to (1) ignoring the fact that the higher-order variables will always have to be disaggregated to the level at which the observations are made, with the problematic statistical consequences that we have described earlier; (2) assuming that the researcher is able to exhaustively identify all the sources of variations at play at the level(s) in question – while experience tells us that this is challenging enough when considering a single level of analysis. Indeed, as noted by Huang & Abdel-Aty: “in the real world …, similar groups may be different in omitted factors and thus may have different means” (2010, p.1559)

In fact, modeling the random variation at the different levels is the most efficient and informative way of “controlling” for the multiple sources of observation dependences that are typically at play in observational studies. , and the view that a researcher can get from the phenomenon he/she investigates can be considerably enriched by even simple ML models (e.g., the variance component models, where only the intercept is allowed to vary).

It is nevertheless important to stress that, while multilevel models offer an elegant solution to the problem of hierarchical data structures, they inherit a good deal of the limitations shown by the regression models from which they are derived. An example is the treatment of correlated predictors. There is, also, a major drawback that is specific of ML models: they can grow very complex very quickly. The number of levels that can be specified is theoretically unlimited, random slopes can be defined at any level for any of the predictors tested… Therefore, it is not only important to check that the application of ML models is indeed warranted (variance partition coefficients, significance of the variances of the random effects…), the responsibility is also left to the researcher to fit parsimonious models, introducing random slopes for particular predictors very sparsely, preferably on the basis of theoretical reasons. In most of the case-studies presented above, for example, random intercepts have been sufficient for correcting the dependence among observations.

6.2. When is it feasible to use multilevel modeling?

The results of the present research confirm the feasibility of applying multilevel models for capturing hierarchical dependences in the observations, resulting from both geographical and accident clustering. However, they also illustrate that this application is much more straightforward in the case of geographical than in that of accident hierarchies.

Geographical higher level effects were found to be significant in almost all types of research question examined, regardless of the geographical unit examined e.g. road sites, regions, countries etc. In most of the cases where accident hierarchy was examined, on the contrary, the results yielded the conclusion that there was no significant variation at the accident or vehicle level. In principle, this means that no substantial correlation is introduced to the data by that type of hierarchical structure, or that the predictors included in the model efficiently tackle the residual variation associated with the higher levels.

However, precisely because of the particular nature of accident data (i.e. many accidents with few vehicles and persons involved in each), it is difficult to tell whether there is indeed no substantial variation at the accident and/or vehicle level, or whether this seems to be the case simply because the number of observations per accident and/or vehicle is insufficient to allow the estimation of the different variance components. In such cases, the choice has been made to leave the model in its single level state, and to perform a standard regression analysis. This choice is motivated by the fact that the particular hierarchical structure of accident data (i.e. many accidents with few vehicles and persons) also leads them to be closer to the structure of independent observations (Lenguerrand et al., 2006)[4].

The results of the present case studies on accident hierarchy also suggest that, in broader and more disaggregate analyses, accident hierarchy effects may be more identifiable. It appears that, focusing on particular groups of road users, or aggregating the characteristics of road users, makes it less feasible to identify higher level random effects. It may be the case, however, that such relationships do exist, but seem to disappear because of the aggregation of different types of road users (“ecological fallacy”).

Figure 5 summarises the theoretical and practical conditions for using multilevel models for analyzing road safety questions. As a general rule, it can be said that multilevel modeling appears to be both more meaningful and

[4] The smaller the lower-to-higher units ratio (number of vehicles per accident, for example), the less serious the underestimation of the standard errors resulting from the fact that the existing correlation between the clustered observations is not taken into account.

easier to apply in the context of macroscopic analyses. On the other hand, the application of multilevel models in the context of microscopic appears to be less straightforward, but not necessarily less critical for the outcomes of the analysis - in several cases, additional insight may be indeed provided by a multilevel model specification.

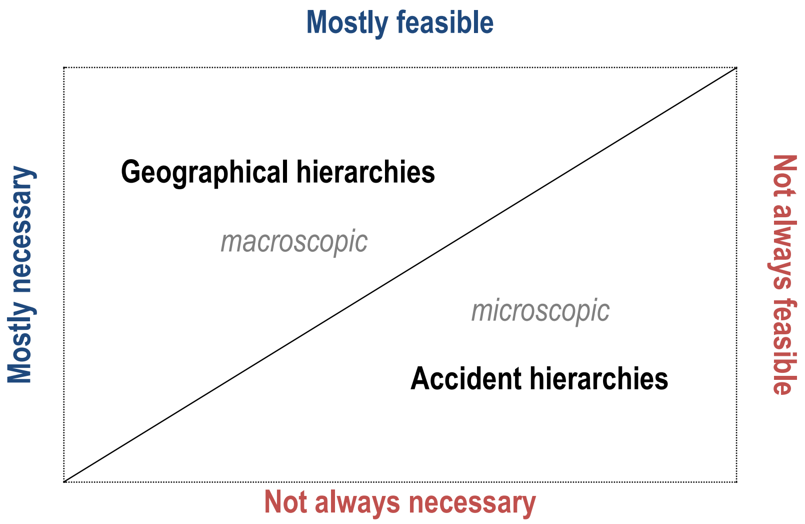


Figure 5. Needs and feasibility of multilevel modeling in road safety research

5.3. The heuristic value of the multilevel framework for road safety research

Although this has been the object of few explicit consideration in previous literature, it is important to insist on the idea that the relevance of ML *formulations* to road safety research is independent of the question of the necessity and feasibility of the application ML *models* to the data. In other words, the possible ML framework underlying a phenomenon also consists of a theoretical framework, whose heuristic value for road safety research is still currentlyunderestimated.

As we saw, the added value of ML models can easily be spoiled if one seeks at fitting models that exhausts the possibilities of the theoretical hierarchy of interest. The *theoretical* ML framework, however, deserves being explored and used as a tool to identify, for example, the levels at which the hypothesized relationships between the predictors and the response variable is likely to be at play, in what form, and under which conditions.

The heuristic value of ML formulations also applies when one is to assess results of previous research. In different studies treating of a given topic, the same dependent variable often appears to be operationalised at different hierarchical level, with potentially important implications for the comparability of the results observed. As an example, Table 3 provides an overview of (single-level) severity analyses. The unit of observations as well as the way the “severity” dependent variable was operationalised clearly indicates that different levels of analysis are at play in the different studies.

Reference	Data Selection	Response Variable	Unit of observation
Evans (1983)	Drivers only	Ratios of killed/surviving drivers	Accident categories based on mass ratios
Chang & Mannering (1999)	All occupants -Truck involved -Non-truck involved	Probability that the most severe injury among occupants will be (1) Prop damage only (2) Possible injury (3) Injury or Fatality	Individual vehicles
Khorashadi et al. (2005)	Drivers only	Driver injury severity (1) None (2) Pain Complaint (3) Visible Injury (4) Severe/Fatal injury	Driver - Vehicle
Shibata & Fukuda (1994)	Fatalities and uninjured only	Fatality risk	Individual Road User
O’Donnell & Connor (1996)	Vehicle occupants only	Occupant injury risk (non-treated injury, treated injury, admitted injury, death)	Individual Road User
Yau (2003)	Single-vehicle accidents only	Probability of most serious injury in crash (Fatal, serious, slight)	Accident
Yau et al. (2006)	Multiple-vehicle accidents (no pedestrians)	Probability of most serious injury in crash (Fatal, serious, slight)	Accident
Kockelman et al. (2002)	- All crashes -2-veh crashes -Single crashes	Probability of different types of injury for driver	Driver - Vehicle
Martin & Lenguerrand (2008)	-2 vehicle crashes -single vehicle crashes -No crashes with only occupants killed or injured	Driver’s vital status	Driver – Vehicle and accident (2 analyses)

Table 3. Overview of the unit of observations and operationalisation of the dependent variable for various accident severity analyses

7. Conclusion

Although multilevel models are commonly applied in many scientific areas, they are relatively new to the field of road safety. This article started on a discussion of the common hierarchical framework characterizing road-safety data, namely geographical and accident hierarchies. This allowed to emphasize the continuity between the two hierarchies, as well as to illustrate the fact that “pragmatic hierarchies” can also be defined to model the internal structure of some observations (multinomial responses, repeated and multivariate measurements).

The general model formulation and basic principles underlying ML techniques were then discussed, and two types of consequences were demonstrated when ignoring a hierarchical structure in the data: statistical and conceptual.

The first consequence is the statistical inaccuracy resulting from the underestimation of standard errors due to the dependence of nested observations. The second consequence is a conceptually impoverished representation of the research topic investigated.

An overview was also provided of the models to be applied to response distributions that are commonly used in road safety, not only to model accident frequency and accident outcomes, but also a host of other road safety indicators. Research applications of these models were also reviewed.

On this basis, the distinction was made between macroscopic and microscopic road safety analyses, both in terms of the statistical need and the feasibility of applying multilevel models.

The advantages and limitations of ML models were finally discussed, and it was stressed that the theoretical/heuristic value of the ML formulation of a research question needs to be considered independently of its practical/statistical applicability.

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Table 1. Sampling models and example link functions for normal responses and for the most current types of discrete responses treated in road safety research.

Distribution of the response variable:	Sampling model (Raudenbusch & Bryk, 2001)	Link function
Normal:	$y_i \sim \mathrm{NID}(\mu_i, \sigma^2)$ E (y_i) = μ_i Var (y_i) = σ^2	- The identity link - $\eta_i = \mu_i$
Bernouilli and Binomial	$y_i \sim \mathrm{B}(m_i, \varphi_i)$ E (y_i) = $m_i \varphi_i$ $\mathrm{Var}(y_i) = m_i \varphi_i (1 - \varphi_i)$	- The logit link- $\eta_i = \log\left(\frac{\varphi_i}{1-\varphi_i}\right)$
Poisson	$y_i \sim \mathrm{P}(m_i, \lambda_i)$ $\mathrm{E}(y_i) = m_i \lambda_i$ $\mathrm{Var}(y_i) = m_i \lambda_i$	- The log link - $\eta_i = \log(\lambda_i)$
Multinomial responses	E (y_m) = $n\varphi_m$ $\mathrm{Var}(y_m) = n\varphi_m(1-\varphi_m)$ $\mathrm{Cov}(y_m, y_{\hat{m}}) = -n\varphi_m\varphi_{\hat{m}}$	- The logit link – $\eta_j = \log\left(\frac{\varphi_j}{\varphi_l}\right)$ (with l being the reference category)