

# Estimating the Conflict-crash Relationship

## Towards Traffic Conflicts Analysis Suitable for Safety Management Practice

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# Introduction

Two general applications of traffic conflicts:

- (1) Quick assessment of crash risk and countermeasures effectiveness (safety engineers)
- (2) Safety effects analysis using statistical models (safety researchers)

This presentation provides the ML-based connection of traffic conflicts to crashes and discusses its estimation suitable for safety engineers.

# Introduction

Failure-caused traffic events include conflicts and crashes. During a conflict, crash does not occur but is uncomfortably close.

Let  $x$  be Lomax-distributed *response time* measured from the moment when crash is too close be acceptable to the moment of evasive action.

The corresponding tail (exceedance) distribution of response time is:

$$\bar{F}(x|k, \theta) = 1 - F(x|k, \theta) = (1 + \theta x)^{-k}$$

Let  $n$  be the number of failure-caused traffic events when time-to-collision is shorter than acceptable  $x_c$ .

The expected number of crashes associated with  $n$  events is:

$$Q_c = n \cdot \bar{F}(x_c) = n \cdot (1 + \theta x_c)^{-k}$$

At crash, response time  $x$  is truncated at  $x_c$  and  $x$  is not observable in its entirety.

Let  $c$  be the number of crashes.

# Introduction

## OLS Estimate of k Parameter

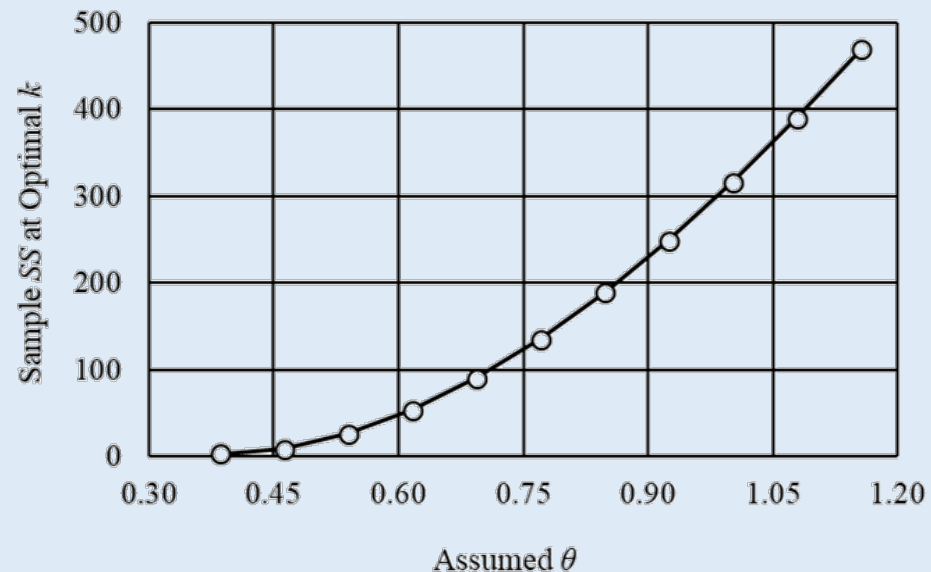
Sort  $n$  response times  $x_i$  in the increasing order.

Calculate  $\bar{F}(x_i) = 1 - (i - 0.5)/n$ .

Take  $\ln$  of both sides of equation:  $1 - (i - 0.5)/n = (1 + \theta x_i)^{-k}$ .

$$\underline{-\ln[1 - (i - 0.5)/n]} = \boxed{k} \ln(1 + \overset{\text{assumed}}{\theta} x_i) \quad i = 1 \dots (n - c)$$

? OLS estimate of  $k$  is:  $\hat{k} = \frac{-\sum_{i=1}^{n-c} \ln(1 - (i - 0.5)/n) \ln(1 + \theta x_i)}{\sum_{i=1}^{n-c} [\ln(1 + \theta x_i)]^2}$



Assume  $\theta = 1/x_c$  ?

$$\hat{Q}_c = n \cdot 2^{-\hat{k}}$$

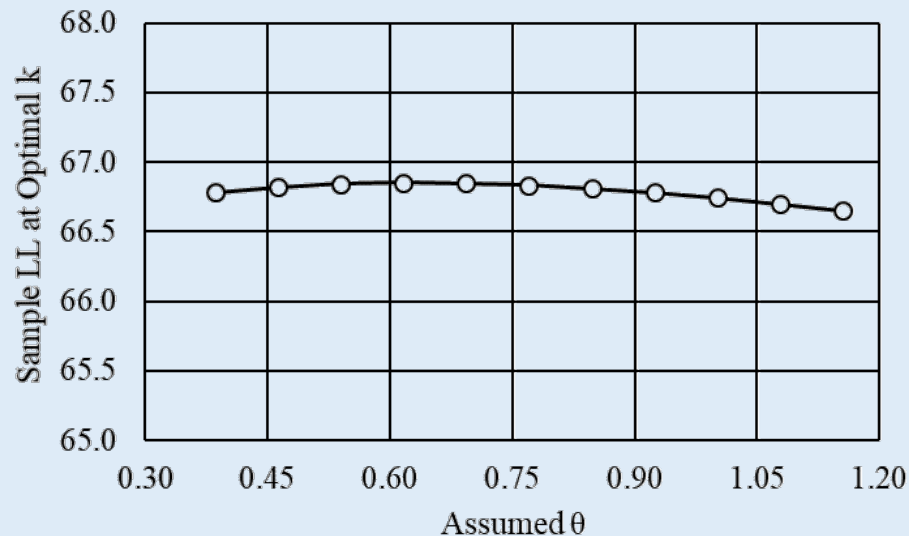
# Three Points to Be Addressed

1. ML estimation of  $k$  parameter
2. Sensitivity of  $k$  and  $Q_c$  estimates to assumed  $\theta$  parameter in single-parameter estimation
3. Two-parameter estimation of Lomax

# ML Estimate of k Parameter

$LL$  for  $c$  crashes and  $(n - c)$  traffic conflicts under  $x_c$  threshold is:

$$LL = \overbrace{(n - c) \ln(\theta k) - (k + 1) \sum_{i=1}^{n-c} \ln(1 + \theta x_i)}^{\text{traffic conflicts}} - \overbrace{ck \ln(1 + \theta x_c)}^{\text{crashes}}$$



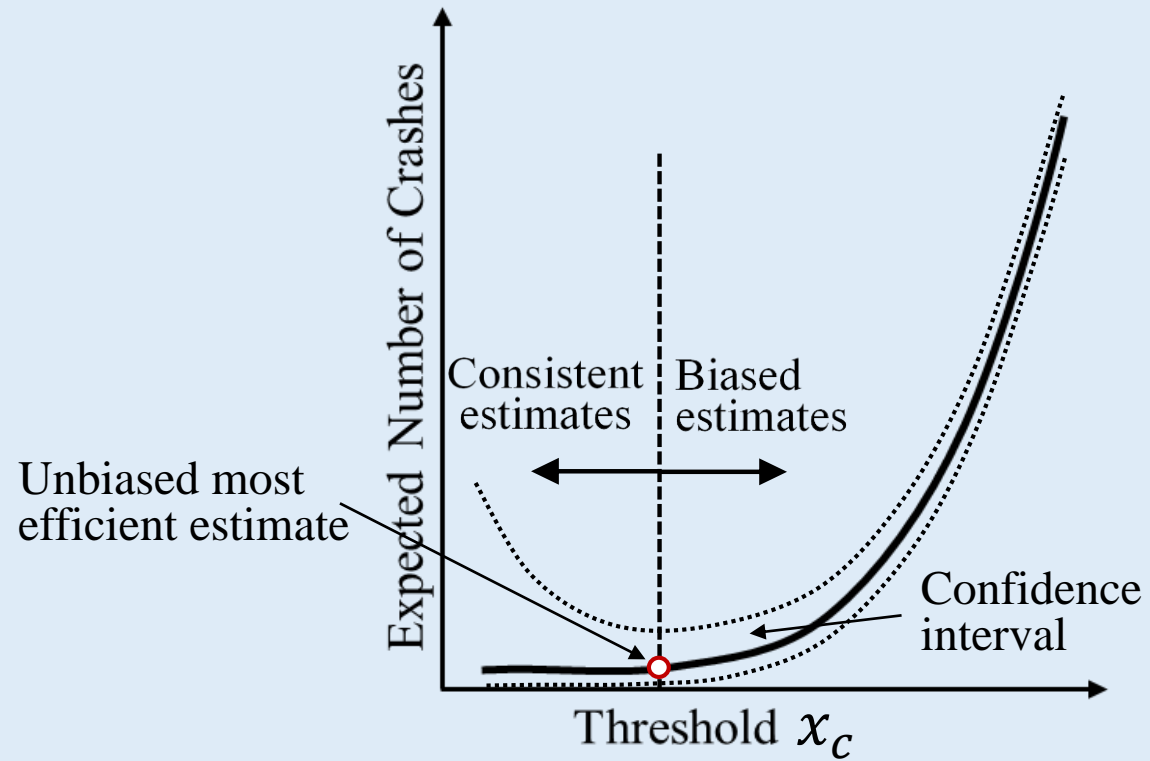
$$\frac{\partial LL}{\partial k} = 0 \quad \text{yields:}$$

$$k = \frac{n - c}{\sum_{i=1}^{n-c} \ln(1 + \theta x_i) + c \ln(1 + \theta x_c)}$$

For  $\theta = 1/x_c$ :

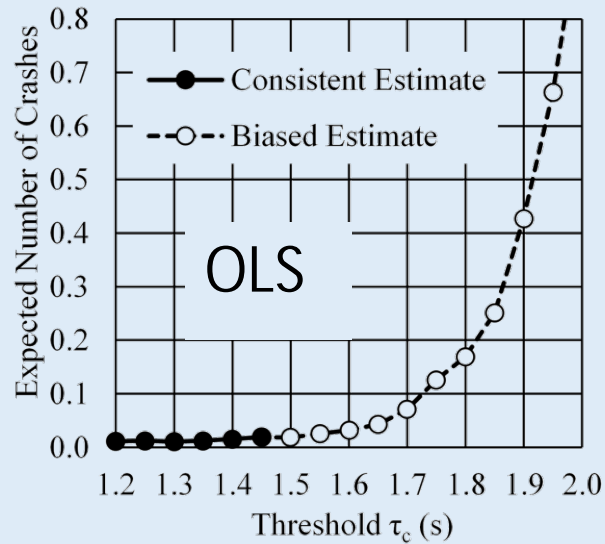
$$k = \frac{n - c}{\sum_{i=1}^{n-c} \ln(1 + x_i/x_c) + c \ln(2)}$$

# Selection of Threshold $x_c$

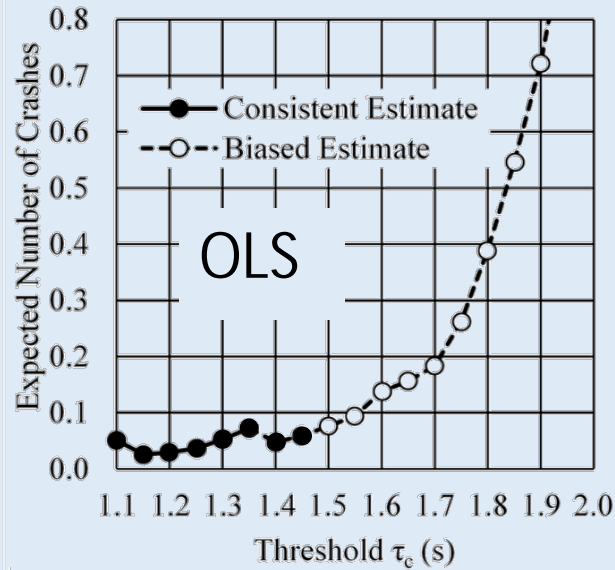


# Expected Crashes Profiles

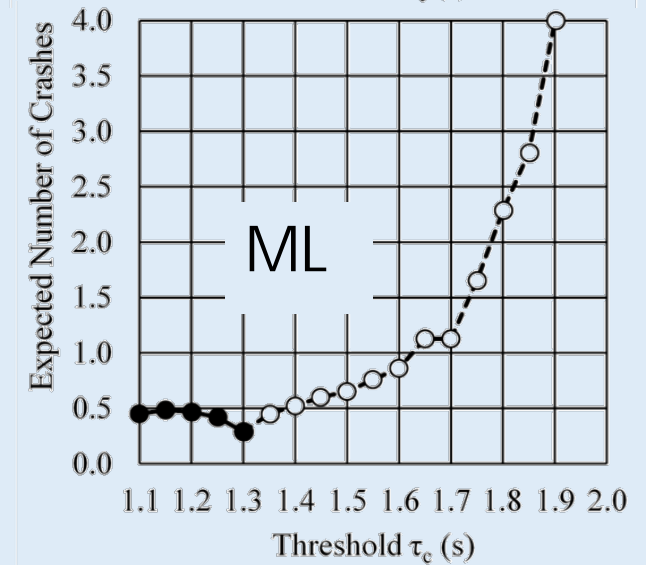
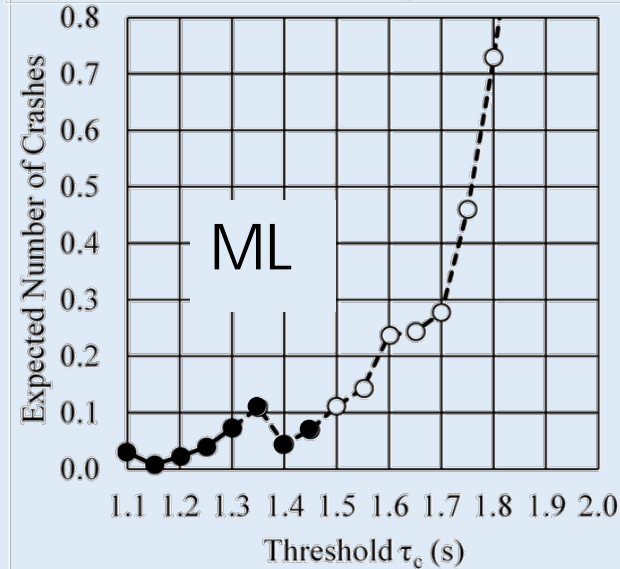
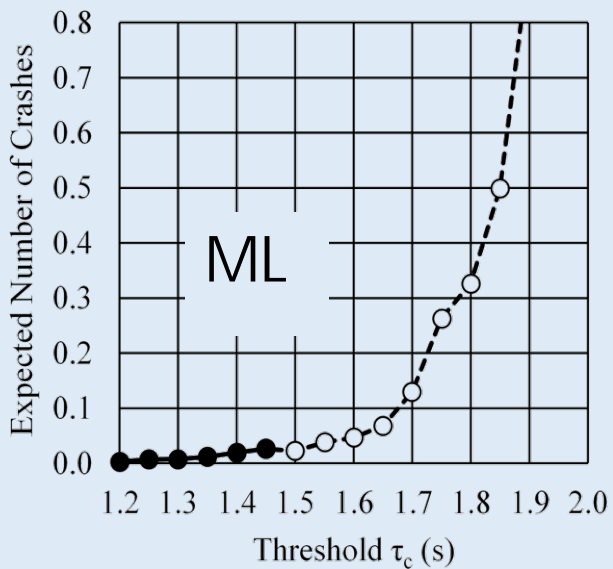
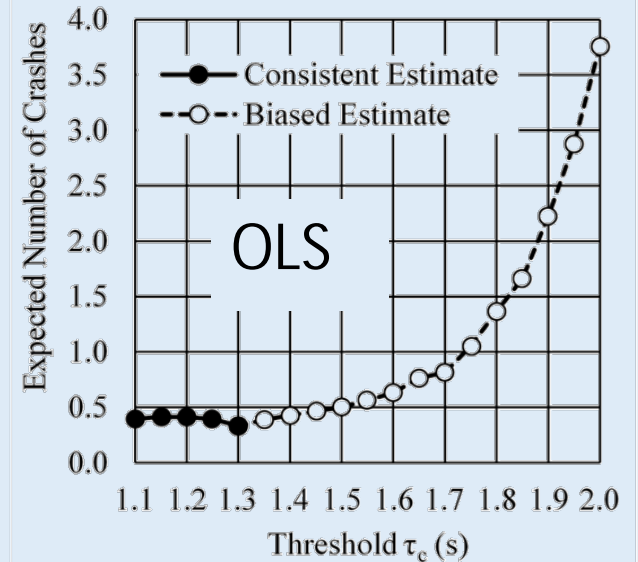
Female drivers 45-64



Male drivers 45-64



Male drivers 16-24





# Comparison of OLS and ML Estimates

## Rear-end Collisions in of SHRP2

Driver Category	Method	Threshold $x_c$	No. of Conflicts	Parameter k	Prob. of Crash Given Conflict $PrC$	Expected Crashes
Female 45-64	OLS	1.45	47	11.257	0.000408	<b>0.0192</b>
	ML	1.45	47	10.797	0.000562	<b>0.0264</b>
Male 45-64	OLS	1.45	56	9.885	0.001057	<b>0.0592</b>
	ML	1.45	56	9.647	0.001247	<b>0.0699</b>
Male 16-24	OLS	1.25	52	7.040	0.007601	<b>0.3953</b>
	ML	1.25	52	6.955	0.008059	<b>0.4191</b>

Note: The values of threshold  $x_c$  were determined with a higher precision than in the original publication (Tarko, 2020).

# Comparing ML and OLS Estimates

## Simulation Experiments

- The Lomax distributions obtained for the SHRP2 drivers were used in the experiments
- Three sources of variability in crash estimates were studied:
  - 1) Arbitrary selection of parameter  $\theta$  (range 0.38 – 1.15)
  - 2) Randomness (90% conf. interval length)
  - 3) Driver types (male 16-24, female 45-64)

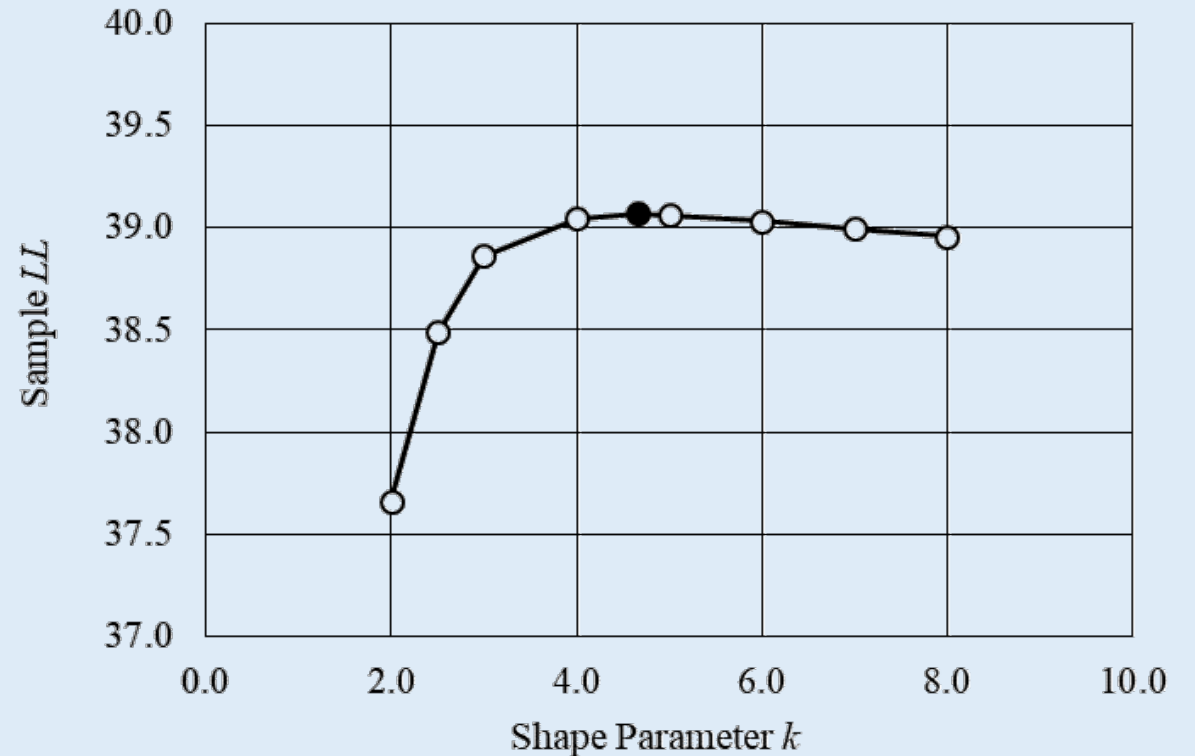
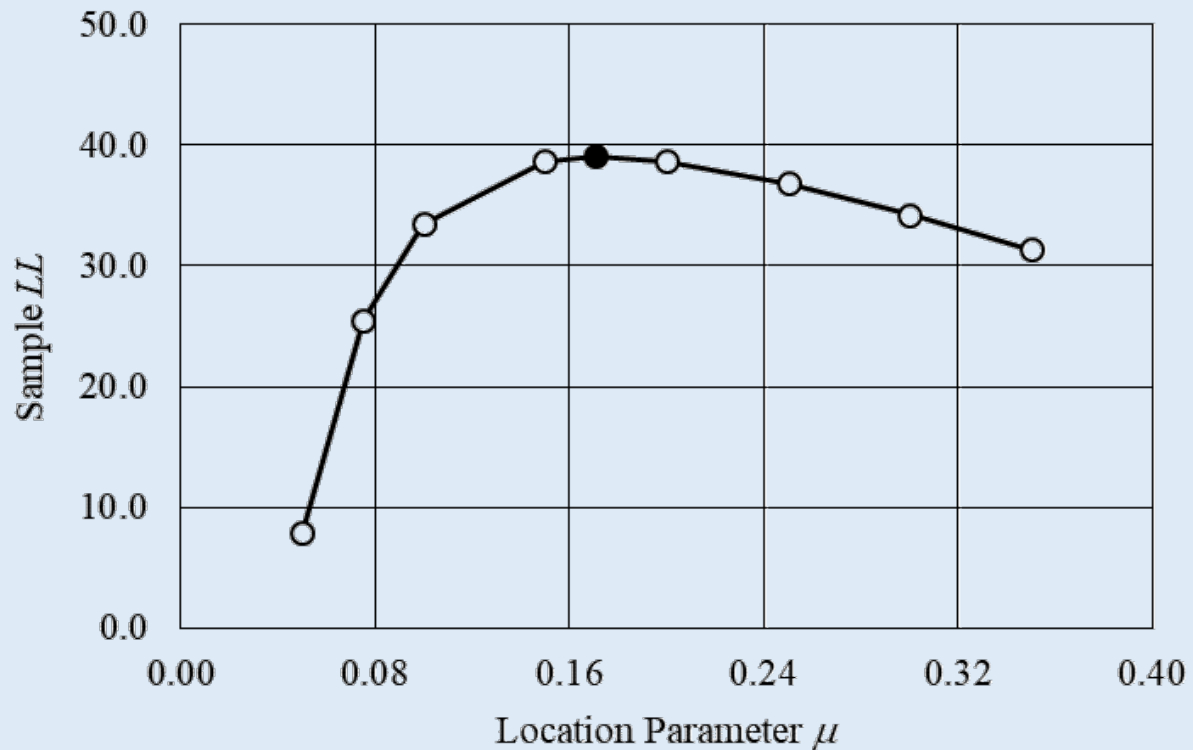
# Variability in Expected Number of Crashes Estimates $Q_c$ (spread)

Source of $Q_c$ Estimate Variability	$Q_c$ Spread = $Q_{c,max} - Q_{c,min}$		Notes
	OLS Estimate	ML Estimate	
<b>Parameter <math>\theta</math> selected from [0.385, 1.155]</b>	0.673 – 0.390 = <b>0.283</b>	0.709 – 0.298 = <b>0.411</b>	Average from 200 trials to control randomness
<b>Randomness (90% confidence interval)</b>	1.024 – 0.163 = <b>0.861</b>	1.007 – 0.153 = <b>0.854</b>	Sample of 100 conflicts
<b>Driver type (male 16-24 vs. female 45-64)</b>	0.527-0.067 = <b>0.460</b>	0.494 – 0.062 = <b>0.432</b>	Fixed exposure (50,000 miles of car following)

# Reparametrized Lomax Distribution

Altun (2021)

$$\theta = \frac{1}{\mu(k-1)} \quad f(x) = \frac{k}{\mu(k-1)} \left( 1 + \frac{x}{\mu(k-1)} \right)^{-k-1}$$



# Concluding Remarks

Neither ML nor OLS methods are efficient in estimating both the Lomax parameters. This finding concurs with the published research.

The existing OLS and ML single-parameter methods with  $\theta$  set at  $x_c$  seems to be acceptable for estimating the crash frequency.

The reparametrized Lomax distribution seems to be promising but estimates of the shape parameter  $k$  exhibit strong variance. More research is needed.

Currently, estimation efficiency is more important than estimation consistency. Massive data potentially available in the future may increase the importance of the estimation consistency.

This presentation did not discuss accounting for heterogeneity via regression analysis to avoid its estimation complexity.

Thank you