Crest Vertical Curvature Safety Assessment through Variable Grade Stopping Sight Distance Control

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Abstract

Stopping Sight Distance (SSD) is a vital design element, which directly affects the control values of critical road design parameters. Although grade marginally affects SSD values, most design guidelines disregard the grade impact during vehicle braking on variable grades. The paper investigates possible deficiency of this approach, regarding cases where the length of the vertical curve exceeds the control SSD values. The authors addressed the SSD calculation on variable grades during the braking process through a recently developed process that relates the point mass model and the laws of mechanics. The paper investigates the adequacy of the American AASHTO (2018), the Greek OMOE-X (2001) and the Italian DM, n.6792 (2001) design guidelines, during the determination of crest vertical curvature rates, from the variable grade SSD calculation point of view for rural arterials and freeways. The assessment was performed for high design speed values of 80km/h, 100km/h and 120km/h respectively, where for each case amended crest vertical curvature rates are delivered as a function of the crest vertical curve’s exit grade value, aiming to grant SSD adequacy throughout the braking process. The analysis, excluding the Greek OMOE-X design guidelines where costly overdesigned control crest vertical curvature rates are adopted, for AASHTO (2018) and DM, n.6792 (2001), revealed a SSD inadequacy on the downgrade area with exit grade values grades below -2% approximately.

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1. Introduction

Based on the American Association of State Transportation Officials [AASHTO, (2018)] road design guidelines, sight distance is the length of roadway ahead that is visible to the driver. The minimum sight distance known as Stopping Sight Distance (SSD), is a highway geometric design element of fundamental importance. SSD must be provided at every point along the road surface, thus affecting critical road design parameters which directly impose economic considerations on both new road designs as well as road improvement projects [e.g. (AASHTO, 2018), (OMOE-X, 2001), (DM. n.6792, 2001)].

The AASHTO (2018), design guidelines (commonly referred to as the Green Book), note that for vertical curves, the grade effect is somewhat balanced and there is no need to adjust SSD due to grade. As a result, in the Green Book (2018) it is stated that the minimum lengths of crest vertical curves, based on sight distance criteria, are generally satisfactory from the standpoint of safety, comfort and appearance, implying that the vertical curvature rate is adequately determined for the suggested grade control values, at least regarding normal design cases and eliminating areas such as decision areas (e.g. ramp exit gores etc.). In other words, although there seems to be a significant difference in SSD values between upgrades and downgrades, the minimum recommended (control) crest vertical curvature rates are determined assuming leveled (0%) grade.

Under this scope, the objective of the present paper is to investigate, from the grade point of view, potential deficiency of this approach, in terms of determining control crest vertical curvature rates based on the actual SSDs as extracted from variable grades.

This assessment was carried out for certain, rather high, design speed values (80km/h, 100km/h and 120km/h), and three different approaches; namely the US (AASHTO, 2018), the Greek (OMOE-X, 2001) and the Italian (DM, n.6792, 2001) design guidelines, regarding the critical case where the length of the vertical curve exceeds the demanded SSD values.

2. Background

According to the existing practice, the SSD of a vehicle consists of two distance components: the distance travelled during driver’s perception – reaction time to the instant the brakes are applied and the distance while braking to stop the vehicle. For example, the SSD model adopted by most design guidelines is represented by Equation (1).

\[
SSD = \frac{V_o}{3.6} + \frac{1}{3.6} \cdot g \int_0^V f_T \frac{V - s}{g} \frac{V_o^2}{m} \, \mathrm{d}V
\]

where:
- SSD (m): stopping sight distance
- \(V_o\) (km/h): vehicle initial speed
- \(t_p\) (sec): driver’s perception – reaction time [2.5sec (AASHTO), 2.0sec (OMOE-X), 2.8-0.01V_o (DM, n.6792)]
- \(f_T\): upper limit of longitudinal friction engaged for braking
- \(A_d\) (N): vehicle drag resistance
- \(m\) (kgr): vehicle mass
- \(s\) (%): road grade [(+) upgrades, (-) downgrades]
- \(g\) (m/sec^2): gravitational constant [9.81m/sec^2]

In AASHTO (2018) design guidelines, the sum of the vehicle’s longitudinal friction and drag resistance is expressed by an average deceleration rate of \(a_{\text{average}}=3.4\text{m/sec}^2\), and the following equation applies:

\[
SSD = \frac{V_o}{3.6} + \frac{V_o^2}{2 \cdot 3.6^2 (a_{\text{average}} + \frac{g}{100})}
\]

where:
- \(a_{\text{average}}\) (m/sec^2): average vehicle deceleration rate [3.4m/sec^2 (AASHTO), see Table 1 for (OMOE-X, DM, n.6792)]

In the Greek (OMOE-X, 2001) and Italian (DM, n.6792, 2001) design guidelines, since the vehicle drag resistance \(A_d\) is speed dependent, SSD values are defined from the integration of equation (1), as seen in Table 1. This means
that an average deceleration rate can be defined on the resulted SSD value from equation (1). This value, extracted for the three examined initial speed values, is also shown in Table 1.

Table 1. Average deceleration rate for initial speed values, based on DM, n.6792 (2001) and OMOE-X (2001) design guidelines.  

<table>
<thead>
<tr>
<th>Design Guideline</th>
<th>$A_d$ / mg (Formula)</th>
<th>$A_d$ / mg</th>
<th>$u_{average}$ (m/sec$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>80km/h</td>
<td>100km/h</td>
</tr>
<tr>
<td>OMOE-X (2001)</td>
<td></td>
<td>2.52 $10^4$V$^2_o$ ($V_o$: km/h)</td>
<td>0.02</td>
</tr>
<tr>
<td>Motorway &amp; Rural Arterial</td>
<td></td>
<td>2.66 $10^4$V$^2_o$ ($V_o$: km/h)</td>
<td>0.02</td>
</tr>
<tr>
<td>DM, n.6792 (2001)</td>
<td></td>
<td>2.66 $10^4$V$^2_o$ ($V_o$: km/h)</td>
<td>0.02</td>
</tr>
<tr>
<td>Motorway</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rural Arterial</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Current road design guidelines define minimum lengths of crest vertical curves as well as the consequent rate of vertical curvature based on SSD provision. Equation (3) and equation (4) illustrate the parameters utilized in determining the length of crest vertical curves ($L$), where the crest vertical curvature rate definition ($H_K$) is shown through equation (5). Based on AASHTO (2018), the values of $H_K$ derived for $SSD \leq L$, apply without significant difference also for the case $SSD>L$.

$$L = \frac{(s_2-s_1)SSD^2}{200(\sqrt{h_1^2+\sqrt{h_2}})^2} \quad \text{SSD \leq L} \quad (3)$$

$$L = 2SSD - \frac{200(\sqrt{h_1^2+\sqrt{h_2}})^2}{s_2-s_1} \quad \text{SSD \geq L} \quad (4)$$

$$H_K = \frac{100L}{s_2-s_1} \quad (5)$$

where:

$H_K$ (m): crest vertical curvature rate
$L$ (m): length of vertical curve
$h_1$ (m): driver eye height [1.08m (AASHTO), 1.06m (OMOE-X), 1.10m (DM, n.6792)]
$h_2$ (m): object height (m) [0.60m (AASHTO), SSD tan(5/60°) (OMOE-X), 0.10m (DM, n.6792)]
$s_1$, $s_2$ (%): grade values

Most of the current efforts to evaluate SSD adequacy are based on 2-Dimensional models. Moreover, as stated by Hassan et al. (1997), such efforts present a fragmented approach (e.g. examination of single elements) in investigating the adequacy of SSD and may underestimate or overestimate the available sight distance and thus possibly lead to safety violations.

The use of the vertical profile is a common approach in determining adequacy of SSD and typically, roadway geometry is evaluated to ensure proper SSD requirements. This approach however fails to examine the continuity of the vertical alignment especially in crest curves and their exiting grades. This was noted in previous research as a potential safety issue [Mavromatis et al. (2012), Moreno et al. (2010)].

There has been very little work on this topic even though there is the potential for requiring different lengths for vertical crest curves when exit grades are considered [Mavromatis et al. (2016)]. For example, according to previous research, a mathematical integration procedure was developed by Thomas et al. (1998), further improved by Taigianidis et al. (2001) and generalized for both crest and sag vertical curves by Hassan (2003), based on the average grade over the braking distance.

From equation (3) and equation (5), it can be seen that the delivered crest vertical curvature rate is not grade dependent. The grade effect is indirectly introduced from the SSD determination, where regarding the majority of road design guidelines worldwide, level road surface is assumed ($s=0\%$).

In various design guidelines, although the same equations are utilized, the adoption of control crest vertical curvature rates is addressed through a number of considerations.
For example in OMOE-X (2001), in cases of two-lane rural roads and motorways, a +10km/h and +20km/h safety margin in the SSD calculation is introduced respectively. Therefore, in order to determine the crest vertical curvature rate for the case of 60km/h design speed on rural arterials, the SSD value utilized refers to 70km/h.

Based on the above equations, Table 2 illustrates, for the assessed design speed values, the adopted design control values (rounded values) in the 3 examined design guidelines regarding SSD and crest vertical curvature rates respectively. It should be noted here that these values are reflective of daylight conditions, since in most design policies guidance is provided for daylight conditions excluding however very few cases (e.g., sag curves).

<table>
<thead>
<tr>
<th>Design Guideline</th>
<th>SSD (m)</th>
<th>Hk (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80km/h</td>
<td>100km/h</td>
</tr>
<tr>
<td>AASHTO (2018) Motorway</td>
<td>130</td>
<td>185</td>
</tr>
<tr>
<td></td>
<td>Motorway</td>
<td>170</td>
</tr>
<tr>
<td>OMOE-X (2001) Motorway</td>
<td>140</td>
<td>205</td>
</tr>
<tr>
<td>OMOE-X (2001) Rural Arterial</td>
<td>95</td>
<td>135</td>
</tr>
</tbody>
</table>

The work presented here regarding the evaluation of the effect of the variable grade during the braking process is based on a recently developed practice (Mavromatis et al., 2012) briefly presented below.

Simple considerations based on the laws of mechanics through equation (6) and equation (7) were applied, assuming time steps of 0.01sec, in order to determine both the instantaneous vehicle speed and pure braking distance (SSD minus distance travelled during driver’s perception-reaction time).

\[ V_{i+1} = V_i - g \left( \frac{a_{\text{average}}}{g} + s \right) t \]  
\[ \text{BD}_i = V_i t - \frac{1}{2} g \left( \frac{a_{\text{average}}}{g} + s \right) t^2 \]  
where:
- \( V_i \) (m/sec): vehicle speed at a specific station i
- \( V_{i+1} \) (m/sec): vehicle speed reduced by the deceleration rate for \( t = 0.01\text{sec} \)
- \( t \) (sec): time fragment \( t = 0.01\text{sec} \)
- \( \text{BD}_i \) (m): pure braking distance

By applying equation (6) and equation (7) subsequently there is a sequence value \( i=k-1 \) where \( V_k \) becomes equal to zero. The corresponding value of \( \Sigma \text{BD}_{k-1} \) represents the total vehicle pure braking distance for the initial value of vehicle speed. The variable grade SSD is produced by adding the final pure braking distance to the distance travelled during the driver’s perception – reaction time [first component of equation (2)] as follows:

\[ \text{SSD} = V_o t_{pr} + \Sigma \text{BD}_{k-1} \]  
where:
- \( V_o \) (m/sec): vehicle initial speed
- \( \Sigma \text{BD}_{k-1} \) (m): total vehicle pure braking distance for the initial value of vehicle speed

Summarizing the SSD determination on variable grade values, the formula shown in equation (2) is used, enriched by the actual grade value portions.

In terms of correlating the present SSD variable grade calculations procedure to the relevant found in the literature [Thomas et al. (1998), Taigianidis et al. (2001) and Hassan (2003)], it was found that the differences never exceeded
±0.70m. However, the proposed approach is based on a selected time step (0.01sec) which compared to the average grade over the braking distance is more accurate.

3. Crest Vertical Curvature Rate Adequacy Investigation

The potential inadequacy of the examined design guidelines regarding the suggested vertical curvature rates, must be sought in the negative grade area since on one hand downgrades increase a vehicle’s SSD and on the other, the current vertical curvature rate definition, for most guidelines is extracted assuming flat vertical geometry.

In the following example, an investigation regarding the crest vertical curvature rate sufficiency of AASHTO (2018) design guidelines is carried out, assuming 80km/h design speed, where the actual SSD values are being defined along two specified positions. Figure 1 illustrates the length of the consequent vertical curve adopted by the Green Book (H<sub>K</sub>=2600m), where the approach and exit grade values were set to +7% and -7% respectively. Two cases of vehicle braking are shown:

Case 1: the braking procedure begins at the starting point of the vertical curve (s=7%) where SSD=121.3m
Case 2: the braking procedure begins at the midpoint of the vertical curve (s=0%) where SSD=136.8m

For both cases, the SSD definition was based on the calculation procedure described previously. As expected, the SSD value in Case 2 is greater not only from the relevant SSD value in Case 1, but also from the SSD design control value of 130m in Table 2 (V=80km/h). In other words, for every SSD value exceeding 130m the calculated crest vertical curvature rate exceeds the control value of H<sub>K</sub>=2600m.

![Fig. 1. Discriminating case of SSD variation on crest vertical curvature rate suggested by AASHTO [V=80km/h, H<sub>K</sub>=2600m].](image)

However, it must be stressed that Case 2 is not the most unfavourable area of excessive SSD value along the vertical curve. The most increased SSD value is delivered at the ending area of the curve and more specifically at the point where the vehicle’s full stop position coincides with the end of the vertical curve (St. 364.00). In general, Figure 1 indicates that there are areas along the vertical curve where the braking procedure requires greater SSD values and thus an increase of the crest vertical curvature rate in these cases seems indispensable.

Figure 2 illustrates control crest vertical curvature rates as calculated from the above SSD assessment for AASHTO (2018), OMOE-X (2001) and DM, n.6792 (2001) design guidelines, referring to V=80km/h design speed. Moreover, it can be seen that for each design guideline, both cases of rural arterials and freeways are examined. For every case, the resultant crest vertical curvature rates are defined through two different approaches; the control values per design guideline (scattered line), and the calculated (modeled) values (continuous line).

More specifically, the calculated H<sub>K</sub> values of Figure 2 were drawn arranging the pure braking process to be performed entirely inside the crest vertical curve. Therefore, the horizontal axis of Figure 2 shows the grade value...
within the crest vertical curve where the vehicle is supposed to terminate the braking performance and is referred as ending (exit) grade value.

The grade values of the horizontal axis in Figure 2 are in line with the suggested control grade values per design speed and road classification by the assessed design guidelines.

Commenting further on Figure 2, according to AASHTO guidelines, when the ending grade of a crest vertical curve is set to the breakpoint of -6% where both rural arterials and freeways seem pertinent, in order to grant SSD adequacy, the minimum crest vertical curvature rate value must be approximately $H_K=2900m$, as opposed to the currently suggested $H_K=2600m$. Respectively, according to DM, n.6792 (2001) guidelines for rural arterials, for ending grade values below -2%, the current value of $H_K=3300m$ seems inadequate.

![Fig. 2. Suggested crest vertical curvature rates based on SSD adequacy (V=80km/h).](image)

Contrary to the above findings, there seems no necessity for the Greek OMOE-X design guidelines to increase the control crest vertical rates of $H_K=4500m$ and $H_K=6200m$ for rural arterials and freeways respectively. The reason is that during the SSD calculation, besides the more conservative object height values utilized, there is a safety margin of +10km/h and +20km/h respectively.

As a rule, the area where the suggested curvature rates (calculated) overlap the respective control rates is defined as critical, since the calculated SSD values exceed the ones utilized by the design guidelines. The intersection between the calculated crest vertical rates and the respective control ones deliver the critical grade value up to which the control crest vertical rates apply.

In order the above procedure to be more integrated, Figure 3 and Figure 4 illustrate the suggested crest vertical curvature rates based on the vertical curve’s ending grade value for design speed values of 100km/h and 120km/h. These figures assess the braking effect on steep (mostly) variable downgrades and thus deliver ready-to-use crest vertical curvature rate values for designers.

The above analysis, for the examined speed values of 80km/h, 100km/h and 120km/h of rural arterials and freeways revealed a SSD inadequacy on the downgrade area with grade values grades below -2% approximately, when control crest vertical curvature rates based on AASHTO (2018) and DM, n.6792 (2001) design guidelines are utilized.
Fig. 3. Suggested crest vertical curvature rates based on SSD adequacy (V=100 km/h).

Fig. 4. Suggested crest vertical curvature rates based on SSD adequacy (V=120 km/h).
However, the Italian DM, n.6792 (2001) design guidelines for both freeways and rural arterials there seem to be mostly affected in terms of crest vertical curvature inadequacy. This is because an older practice for object height is adopted (0.10m) and not the current trend according to which the object height is equivalent to the tail lights (0.50m or 0.60m), as adopted by many design guidelines worldwide. If 0.50m are applied as object height, at least for downgrades up to -6% the current values of crest vertical curvature rates seem sufficient for all the examined cases.

4. Conclusions

The paper investigates the adequacy of AASHTO (2018), DM, n.6792 (2001) and OMOE-X (2001) design guidelines, during the determination of crest vertical curvature rates, from the variable grade SSD calculation point of view for rural arterials and freeways. The assessment was performed for high design speed values of 80km/h, 100km/h and 120km/h respectively.

Initially, the authors addressed the SSD calculation on variable grades during the braking process through an earlier approach based on the point mass model and the laws of mechanics. The resultant crest vertical rates apply for cases where the length of the vertical curve exceeds the calculated SSD values.

Excluding the Greek OMOE-X design guidelines, where costly overdesigned crest vertical curvature rates are adopted, regarding AASHTO (2018) and DM, n.6792 (2001), the analysis revealed a SSD inadequacy on the downgrade area with grade values grades below -2% approximately. Therefore, the authors provided amended crest vertical curvature rates, in order to grant SSD adequacy throughout the braking process.

Especially for the Italian DM, n.6792 (2001) design guidelines for both freeways and rural arterials, the existing values at first glance seem vastly inadequate. However, by increasing the object height to 0.50m, at least for downgrades up to -6% the current values of crest vertical curvature rates seem sufficient for all the examined cases.

An immediate implementation of the present approach is to provide the designers with ready-to-use revised crest vertical curvature rates, based on the desired exiting grade value of the design and in accordance to roadway’s functional classification.

However further analysis is required in order to include the effect of combined horizontal – vertical alignment, certain arrangements of which might impose additional restrictions. Additional qualitative research seems necessary as well aiming to evaluate parameters of SSD (braking on curves, ABS braking, friction coefficient etc.), reflect current vehicle dynamics trends and thus simulate the braking procedure more realistically. Finally, one should not ignore the fact that the human factor might impose additional restrictions and consequently influence the braking process to some extent beyond the perception-reaction procedure and friction reserve utilized in the braking process.

References


